

Information, Commitment, and Separation in Illiquid Housing Markets*

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Abstract

I propose a model of the housing market using a search framework with asymmetric information in which sellers are unable to commit to asking prices announced ex ante. Relaxing the commitment assumption prevents sellers from using price posting as a signalling device to direct buyers' search. Adverse selection and inefficient entry on the demand side then contribute to housing market illiquidity. Real estate agents that can facilitate the search process can segment the market and alleviate information frictions. Even if one endorses the view that real estate agents provide no technological advantage in the matching process, incentive compatible listing contracts are implementable as long as housing is not already sufficiently liquid. The theoretical implications are qualitatively consistent with the empirical observations of real estate brokerage: platform differentiation, endogenous sorting, and listing contract features that reinforce incentive compatibility.

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1 Introduction

In this paper, I develop a search-theoretic model of the housing market that (i) employs a method of price determination that accounts for the strategic interaction between buyers and sellers; (ii) incorporates the documented heterogeneity in seller motivation and asymmetry of information; and (iii) provides insight about the role of real estate agents and intuition for the seemingly puzzling structure of listing contracts. I first show that satisfying the first two requirements leads to an equilibrium with adverse selection and inefficient entry of buyers. I then focus on the potential role of real estate agents in overcoming information frictions and improving market efficiency.

Extensive empirical work has established several stylized facts about housing market prices and selling times.¹ The correlation between prices and liquidity and the observed price dispersion in housing markets point to search theory as an appropriate modelling technique. While existing search models of the housing market can account for a wide range of the empirical trends, I argue that off-the-shelf search frameworks are not consistent with casual observations of the real estate market. For instance, some of these models do not allow for multiple offers by competing bidders, while others ignore the possibility of renegotiating offers announced *ex ante* when there are *ex post* incentives to do so. I show that accounting for these phenomena in the pricing protocol of a search and matching model has implications for liquidity and efficiency, and introduces the informational role of agency in illiquid markets.

There is good reason to suspect that sellers of identical houses differ in terms of their reservation price. Glower, Haurin, and Hendershott (1998) conduct a survey of home sellers and find substantial heterogeneity in terms of motivation to sell: some sellers have a strong desire to sell quickly, while other sellers are much more patient. A seller's degree of patience can be a reflection of a job opportunity elsewhere or the seller's arrangement to purchase

¹See for example, Glower, Haurin, and Hendershott (1998), Merlo and Ortalo-Magné (2004) and Krainer (2001).

her next home (i.e., the seller might have already bought a new home, and wants to sell the first home quickly in order to avoid double mortgage payments). Accordingly, I introduce heterogeneity on the seller side of the market to reflect differences in reservation values. Importantly, the seller's willingness to sell is unobservable to the buyer. Market participants would benefit if this information could be credibly conveyed, for example, by means of list prices. I show that the inability to commit to a list price prevents sellers from using price posting as a signalling device. Instead, patient sellers mimic impatient sellers in order to drive up the final sale price by increasing the probability of a bidding war. Consequently, illiquidity in the housing market is rendered more severe because of adverse selection and inefficient entry on the demand side.

I extend the model to include real estate agents as service providers that can alleviate the burden associated with the process of searching for a home. Agents help buyers find suitable properties and provide expert advice and marketing services to sellers. In North American housing markets, sellers typically pay the real estate commission fees, while much of agents' efforts and services are aimed at facilitating home buying. By modelling the listing agreement between a seller and her agent, I find that in some circumstances, real estate agents can offer incentive compatible contracts to segment the market by seller type. This alleviates the information problem and increases liquidity in the housing market. Even if real estate agents provide no technological advantage in the matching process, incentive compatible listing contracts are implementable as long as housing is sufficiently illiquid; i.e., a house is not readily saleable due to search and information frictions.

In the theory, incentive compatibility does not require real estate agent involvement in the pricing mechanism or access to a commitment technology. Nor does it rely on exogenously imposed assumptions on preferences or technologies to satisfy a Spence-Mirrlees sorting condition, since sellers need not benefit directly from real estate services and the cost is independent of a seller's type. Instead, the housing market is characterized by a directed search environment in which real estate agents play the role of market makers as in Mortensen and Wright (2002). Designing a new real estate listing agreement creates

a new submarket in the search framework that can potentially attract sellers and buyers. Sellers respond differently to changes in the arrival rate of buyers, which in turn is related to the endogenous composition of sellers. Anxious sellers might be willing to spend more on real estate services if it allows them to distinguish themselves from relaxed sellers and attract more potential buyers. Market separation is therefore the result of a sorting condition that arises endogenously because of the beliefs and equilibrium search strategies of buyers. These theoretical predictions are consistent with the recent empirical evidence of endogenous sorting and service differentiation between full-commission full-service realtors, and low-cost limited-service agents (Bernheim and Meer, 2008; Levitt and Syverson, 2008a; Hendel, Nevo, and Ortalo-Magné, 2009): sellers represented by full-commission agents tend to exhibit characteristics consistent with high motivation to sell, and consequently experience shorter selling times and a higher probability of sale.

This paper is related to the recent literature that applies search theory to model the housing market (Wheaton, 1990; Krainer, 2001; Albrecht et al., 2007; Díaz and Jerez, 2010; Head, Lloyd-Ellis, and Sun, 2011). My approach differs from these papers in that I develop a process of price determination that reflects the following: sometimes the terms of sale are determined through bilateral bargaining, other times the house is sold in an auction with multiple bidders. Moving away from Nash bargaining and non-negotiable price posting towards a setting that more closely resembles the pricing mechanism observed in North American real estate markets has important implications for housing liquidity and market efficiency.

The model presented here is perhaps closest to Albrecht, Gautier, and Vroman (2010). They also depart from benchmark search models and allow for multilateral matches with terms of trade determined through auctions. What makes this model different from theirs is that sellers cannot commit to sell when a buyer offers the list price. In Canada and the U.S., there is no such commitment mechanism, at least in the form of a legal obligation associated with the list price that compels a seller to accept an offer. The fully separating and constrained efficient equilibrium obtained in Albrecht, Gautier, and Vroman (2010) is

no longer achievable; by relaxing the commitment assumption, I show that the equilibrium is necessarily pooling. The housing market is then plagued by illiquidity as a result of adverse selection and inefficient entry of buyers relative to a full information economy or the solution to a social planner's problem. By introducing real estate agents, I investigate when separation can be restored and the implications for liquidity and efficiency in the housing market. The results are robust to changes in the fee structure of real estate listing agreements and to different specifications and interpretations of the services provided by real estate agents. In particular, market separation remains feasible when real estate fees are expressed as a flat upfront fee or as a fixed percentage of the sale price, and even in the case where real estate services are only valuable as a potential signalling device and not for other exogenous reasons. In general, sellers signal their willingness to sell via their choice of real estate agent. High fee agents represent anxious sellers and as a result attract more buyers, while relaxed sellers are more likely to list their house without the assistance of an agent, or with limited-service discount realtors.

This paper also contributes to the search literature, and in particular the study of markets with search frictions and private information. With only a few exceptions, most theories rely on strong commitment assumptions. Guerrieri, Shimer, and Wright (2010) present a search environment with adverse selection and show that screening can at least partly alleviate the symptoms of private information in a competitive search environment when the uninformed party can commit to a take-it-or-leave-it trading mechanism. Delacroix and Shi (2013) study a model with adverse selection where sellers can post non-negotiable prices as a means of directing search, and also as a signal of the quality of their asset. In contrast, relaxing the assumption of full commitment to the announced terms of trade is an important element in this paper.

Kim (2012) shows that non-binding messages can generate a partially separating equilibrium in a decentralized asset market when there is private information about the quality of the asset. Sustaining endogenous market segmentation requires interdependent values: the condition that the seller's type affects the buyer's value. Here, the hidden information is

the seller's motivation, which is independent of the buyer's valuation. Menzio (2007) relaxes the commitment assumption in a model of the labour market and shows that cheap talk can sometimes credibly convey information when wages are determined through bargaining. Restricting the process of wage determination to a bilateral bargaining game limits the share of the surplus that can be extracted by a deviating firm. A deviation can improve the matching probability, but will result in a lower negotiated share of the surplus. In my environment with auctions, the transaction price increases with the number of buyers in a match.² Without commitment to the asking price, this hinders truthful information revelation and unravels market separation in the version of the model without real estate agents.

The next section presents the model of the housing market with heterogeneity in seller motivation but without real estate agents. A comparison of the market equilibrium with the constrained efficient allocation leads to a discussion of how information frictions give rise to housing illiquidity. Real estate agents are introduced in Section 3. Section 4 concludes.

2 The Model

There is a fixed number S of sellers, and a number B of buyers determined by free entry. Heterogeneity on the seller side reflects differences in willingness to sell. Consistent with the evidence documented by Glower, Haurin, and Hendershott (1998), some sellers are desperate to sell quickly, while other sellers are more relaxed.³ In a dynamic setting, preferences over price and liquidity would reflect in the discount rate and patient sellers would be more inclined to turn down low offers and wait for more favourable terms of trade in the future. In a

²Julien, Kennes, and King (2006) highlight the implications of this type of setup for residual price dispersion in a theory of the labour market with full information.

³For instance, a seller moving to another city to start a new job is likely willing to sell at a low price if it means a shorter time on the market. On the other hand, a seller hoping to move to a different neighbourhood in the same town is more inclined to hold out for a higher sale price. The fact that most sellers are also buyers in the housing market is likely another source of heterogeneity in seller motivation. Some sellers might have already submitted offers to purchase another home. Illiquidity in the housing market means that they may either find themselves servicing two mortgages, or have the purchase fall through if it was a conditional-on-sale offer.

static setting, heterogeneity in reservation values is sufficient for capturing this phenomenon. A fraction σ_0 of sellers are anxious or impatient with a low reservation value, c_A . The remaining $1 - \sigma_0$ of sellers are relaxed/patient, with a high reservation value, $c_R > c_A$. Differences in sellers' willingness to sell is an important source of asymmetric information in the housing market, since reservation values are unobservable to buyers. Buyers pay a cost κ_0 to enter the market and visit a home listed for sale. Buyers are homogeneous, and assign value $v > \kappa_0 + c_R$ to home ownership. While ex post buyer heterogeneity would add another element of realism to the model without affecting the main results, omitting match specific values reduces the risk of obscuring the central arguments of the paper.⁴

If a buyer meets a seller and a transaction takes place at price p , the payoff to the buyer is $v - p$, and the payoff to the seller is $p - c$, where $c \in \{c_A, c_R\}$ refers to the reservation value of the seller. Each buyer visits exactly one seller, but the matching process is subject to frictions. Specifically, the probability that a seller is matched with exactly k buyers follows a Poisson distribution,

$$e^{-\theta} \cdot \frac{\theta^k}{k!}, \quad k = 0, 1, 2, \dots$$

where θ denotes the ratio of buyers to sellers/houses, or market tightness.⁵ In a directed search setting, there can be multiple submarkets with different buyer-seller ratios. Buyers can condition their entry/search decisions on observable characteristics (such as non-binding list prices), so that tightness in each submarket is determined endogenously.

I depart from the price determination mechanisms typically used in off-the-shelf search models. Nash bargaining is inappropriate for modelling the interaction between buyers and sellers in housing markets with multilateral matches (i.e., when several buyers visit the same

⁴The homogeneous demand side is less general than search models with match-specific values (for example, Albrecht, Gautier, and Vroman, 2010). I show in Appendix B that introducing ex post buyer heterogeneity would not alter the separation and efficiency results. It would, however, add to the analytical complexity when real estate agents are introduced in Section 3.

⁵This matching process often emerges in the literature in models of economies with coordination frictions and a finite number of sellers and buyers with symmetric search strategies. The Poisson matching probabilities are calculated for a large market with $B, S \rightarrow \infty$ and $B/S = \theta$ (see Butters, 1977; Burdett, Shi, and Wright, 2001).

house). Price posting by sellers requires commitment, even though ex post there are incentives for sellers to allow buyers to bid the price up above the posted price. Instead, I propose a different mechanism to reflect these important dimensions of house price determination. In a bilateral match, the buyer negotiates directly with the seller, but if other buyers are interested in the same house, they bid competitively for the purchase.

2.1 Buyers' Bidding Strategies

Consider a housing market characterized by the buyer-seller ratio θ , and the fraction of highly motivated sellers σ . If a buyer is the only one to visit a particular house (a bilateral match), he is free to make an offer without worrying about competing bidders. In such cases, suppose the buyer can make a take-it-or-leave-it offer. The solitary buyer offers either c_A or c_R , whichever yields the highest expected payoff. If $\sigma(v - c_A) > v - c_R$, there is a selection problem, and the buyer offers c_A , knowing that if the seller is of type R , the offer is rejected and there is no transaction. Otherwise, the buyer sensibly offers c_R , and trade will occur regardless of the seller's type. When more than one buyer arrives (a multilateral match), they compete for the house in a private value sealed bid auction.⁶ A potential buyer can observe the number of competing bidders,^{7,8} so that buyers compete à la Bertrand and bid

⁶The theoretical results in this paper are robust to several perturbations of the process of price determination. For example, it is straightforward to show that the expected payoff functions are unaltered when buyers are permitted to submit bids with escalator clauses, or when sellers run simultaneous multiple round auctions.

⁷This assumption is consistent with a survey of recent home buyers, conducted by Genesove and Han (2011). A seller has a vested interest in disclosing this information, since the presence of other buyers escalates the price of her house. There are strategic ways to credibly convey this information to competing bidders. For example, a home listing can specify a date and time when offers will be accepted and reviewed. This leads to a scenario with competing bidders in the same location at the same time, where buyers can condition their bidding strategy on the number of other buyers interested in the same house. In any case, buyers merely need to know that there is at least one other competing bidder. Alternatively, sellers can reveal the initial offers and provide potential buyers with an opportunity to resubmit. Permitting buyers to submit bids with escalator clauses would similarly circumvent the issue of credible disclosure regarding the participation of other bidders (see footnote 6).

⁸In contrast, Kim and Kircher (2012) study the separation and efficiency results in a labour market context when the number of competing bidders is unobservable. They show that full efficiency can be sustained without commitment to reservation prices in a directed search setting with first-price auctions.

their valuation, v . The seller randomly selects among the buyers, so that each bidder has an equal probability of purchasing the home.

This method of price determination is stark in the sense that the price jumps from the take-it-or-leave-it offer to v when the number of bidders increases from one to two. This outcome of the pricing process can be tempered without changing the theoretical implications of the model by, for example, adding idiosyncratic variation in buyer valuations (see Appendix B) and/or assuming that the surplus is split between the buyer and the seller in a bilateral match by incorporating a bargaining game such as the one studied by Grossman and Perry (1986) and used by Menzio (2007). The version of the model presented here has the advantage that it is straightforward to check the incentive compatibility of market separation while maintaining the result that transaction prices are higher in multiple offer situations.

2.2 Expected Payoffs and Free Entry

The expected payoff to the buyer is

$$U(\sigma, \theta) = e^{-\theta} \max \left\{ \sigma(v - c_A), v - c_R \right\} = \begin{cases} e^{-\theta} \sigma(v - c_A) & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\ e^{-\theta} (v - c_R) & \text{if } \sigma \leq \frac{v - c_R}{v - c_A} \end{cases} \quad (1)$$

This is the payoff in a bilateral situation, which occurs with probability $e^{-\theta}$. The expected payoff in a multilateral match with $k \geq 1$ other buyers is zero since the equilibrium bid is v . Two cases arise because the cut-off for offering c_A in a bilateral match depends on the fraction of anxious sellers, as described above. The expected payoff function (1) and the free entry of buyers, $U(\sigma, \theta) = \kappa_0$, determine the equilibrium buyer-seller ratio, θ .

The expected payoff to a relaxed seller is

$$V_R(\sigma, \theta) = e^{-\theta} \sum_{k=2}^{\infty} \frac{\theta^k}{k!} (v - c_R) = [1 - (1 + \theta)e^{-\theta}] (v - c_R) \quad (2)$$

The final expression recognizes the McLaurin series of the exponential function. The simplicity of this expression arises because the payoff to a type R seller in a bilateral match is zero regardless of whether or not a transaction takes place. A motivated seller, on the other hand, has the following expected payoff:

$$V_A(\sigma, \theta) = [1 - (1 + \theta)e^{-\theta}] (v - c_A) + \begin{cases} 0 & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\ \theta e^{-\theta} (c_R - c_A) & \text{if } \sigma \leq \frac{v - c_R}{v - c_A} \end{cases} \quad (3)$$

The last term reflects the positive surplus for a type A seller in a bilateral match whenever the buyer offers $c_R > c_A$. Anxious sellers only benefit from the $c_R - c_A$ surplus in a bilateral match (hereinafter “bilateral bonus”) if $\sigma \leq (v - c_R)/(v - c_A)$.

2.3 Full Information Benchmark

If sellers’ reservation values were observable, buyers could condition their search strategy and bilateral offers on the seller’s willingness to sell. The expected payoffs to sellers in a housing market with observable c_A and c_R , according to (2) and (3), are

$$V_A(1, \theta_A) = [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) \quad (4)$$

$$V_R(0, \theta_R) = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) \quad (5)$$

with $\{\theta_A, \theta_R\}$ determined by the free entry conditions according to (1):

$$U(1, \theta_A) = e^{-\theta_A} (v - c_A) = \kappa_0 \quad (6)$$

$$U(0, \theta_R) = e^{-\theta_R} (v - c_R) = \kappa_0 \quad (7)$$

With full information, the pricing mechanism is efficient in the sense that a house is always transferred to a bidder, and no buyer-seller match leaves positive surplus on the table. Market efficiency of the separating equilibrium further requires that θ_R and θ_A maximize social

surplus. Since buyers face undistorted incentives in searching for a house, this requirement is satisfied in the full information separating equilibrium.

Proposition 2.1. *The decentralized equilibrium under full information is constrained efficient.*

Constrained efficiency means the social planner is also subject to the same search frictions faced by market participants. The proof of Proposition 2.1 shows that the optimality conditions associated with the planner's problem coincide exactly with the free entry conditions for buyers in housing markets with full information. All proofs are relegated to Appendix A.

2.4 Equilibrium and Efficiency Under Asymmetric Information

In contrast to the full information equilibrium, the equilibrium of the model with unobservable reservation values is a random search equilibrium with both types of sellers attracting buyers in a single market. Equilibrium payoffs are given by (1), (2), and (3) with θ determined by a single free entry condition and σ equal to the aggregate fraction of motivated sellers, σ_0 . The information problem generates illiquidity in the housing market due to adverse selection and inefficient entry. Figure 1 illustrates the liquidity of housing (as measured by the average probability of a transaction) in the housing market equilibrium relative to the full information benchmark in terms of the composition of sellers. When σ_0 is high, $\sigma_0 > (v - c_R)/(v - c_A)$, the adverse selection problem is severe in the sense that buyers make take-it-or-leave-it offers in bilateral matches that get rejected whenever the seller is less motivated to sell. Failure to trade in a match even when the surplus is positive reduces the number of transactions in the real estate market relative to the efficient allocation. Even when σ_0 is low, $\sigma_0 \leq (v - c_R)/(v - c_A)$, the private information about the seller's motivation makes housing less liquid. When buyers offer $c_R > c_A$ in a bilateral match and their share of the surplus in a transaction with an impatient seller is reduced, fewer buyers find it worthwhile to participate in the housing market. This is an implication of the free entry condition.

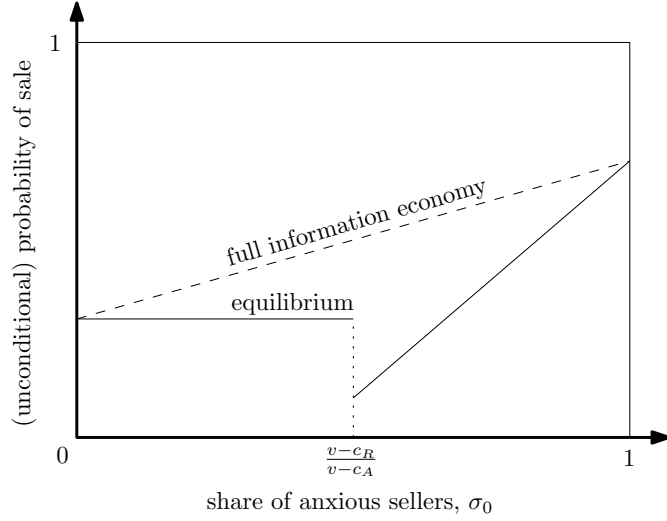


Figure 1: Housing liquidity in equilibrium relative to the full information benchmark.

The full information equilibrium and solution to the social planner's problem establish that it is efficient for sellers with different reservation values to be distinguishable. With c_A and c_R unobservable, there could be efficiency gains associated with a mechanism that allows sellers to reveal their type. If sellers can differentiate themselves, buyers can direct their search. More buyers will visit the impatient sellers, knowing that a lower offer will be accepted in a bilateral match. Past studies have proposed the list price as a means of signalling private information (Albrecht, Gautier, and Vroman, 2010; Delacroix and Shi, 2013). Menzio (2007) shows that non-contractual messages in job listings can sometimes credibly convey information when wages are determined through bilateral bargaining. In my framework, the list price as a non-contractual message is not a credible signalling device: Type R sellers will list their homes to mimic the list prices of type A sellers in order to attract more buyers. This increases the probability that a bidding war will drive the selling price upward. In the event of a bilateral match, a type R seller's payoff is zero regardless of whether the buyer offers c_R (leaving the seller with none of the surplus) or c_A (in which case the relaxed seller simply rejects the offer). This result is stated formally in Proposition 2.2.

Proposition 2.2. *If $\{p_1, p_2\}$ denote any two negotiable list prices announced by house sellers*

and $\{\theta_1, \theta_2\}$ the respective buyer-seller ratios in equilibrium, then $\theta_1 = \theta_2$.

A correlation between the non-binding list price and the seller's reservation value is unsustainable, and the equilibrium reduces to random search with uninformative list prices. Without the ability to commit to list prices, market separation violates incentive compatibility. Note that the incentive for a relaxed seller to mimic a motivated seller is clear in the current setup given the the assumption of take-it-or-leave-it offers. The result in Proposition 2.2, however, obtains in other settings whenever the appeal of a higher expected selling price in multiple offer situations outweighs the possibility of low or even unacceptable offers if fierce enough competition between buyers does not unfold.

Housing units are illiquid in the pooling equilibrium relative to the full information benchmark. This result lies in stark contrast to the usual market separation results in the directed search literature. Even with asymmetric information, access to a technology that allows sellers to commit to a posted price would implement the separating allocation.⁹ As demonstrated by Albrecht, Gautier, and Vroman (2010), even partial commitment to an asking price (specifically, only offers below the posted price can be rejected) is sufficient for achieving constrained efficiency. In Canada and the U.S., however, there is no legal obligation associated with a list price that forces acceptance of an offer by the seller. In the next section, I investigate whether real estate agents can fulfil an informational or signalling role in the housing market. I derive conditions that permit real estate agents to offer distinct incentive compatible listing agreements to segment the market, allow buyers to direct their search, and help overcome the problems related to asymmetric information. It turns out that in some cases, the type of real estate contract that is often observed in housing markets is conducive to market separation.

⁹I do not prove this here, as the market separation and efficiency results with commitment to posted mechanisms are well known in the literature.

3 Real Estate Agents

I add real estate agents to the model as a way of endogenizing κ_0 : the buyer's cost of searching for a house. Intuitively, real estate agents (REAs) have access to more detailed information about the characteristics of houses and the idiosyncratic preferences of prospective buyers. Acquiring and using this knowledge can reduce the informational burden of searching for a home. Detailed listings, databases of relevant real estate information, and advertisements are created to help guide buyers throughout the search process. In addition, REAs work with a sellers to showcase the features of a unit by decluttering, painting, repairing, renovating, decorating, and staging the home. Let $a \in [0, \infty)$ denote the level of services supplied by a REA, and let the search cost be a decreasing function of a , $\kappa : [0, \infty) \rightarrow [0, \kappa_0]$, with $\kappa(0) = \kappa_0$ and $\lim_{a \rightarrow \infty} \kappa(a) = 0$. Of course, providing services to decrease κ is costly for the real estate agent. The cost associated with supplying a given level of service is determined by the function $\phi : [0, \infty) \rightarrow [0, \infty)$. The cost function satisfies the following properties: $\phi(0) = 0$, $\phi'(a) > 0$ for all $a \in [0, \infty)$, and $\lim_{a \rightarrow \infty} \phi(a) = \infty$.

A REA proposes an arrangement $(a, z) \in \mathbb{C}$ to be accepted by a seller: a is the extent of the REA's marketing efforts, which can also be expressed in terms of κ (the cost borne by a buyer that searches among the houses listed with agents providing service level a); z is the REA's commission, expressed as an upfront non-refundable fee; and $\mathbb{C} = [0, \infty)^2$ is the set of all possible listing contracts. The flat fee assumption is made for tractability, and is sufficient for deriving results that are robust to changes in the structure of the REA's commission. Since fixed rate commission structures are also commonly observed in residential real estate markets (Hsieh and Moretti, 2003; Federal Trade Commission and U.S. Department of Justice, 2007), I return to fixed rate contracts in Section 3.4 and show that features common in real world listing contracts are related to incentive compatibility. The market for REAs is assumed to be frictionless and perfectly competitive.

I study the equilibria of the following two stage game: in the first stage, REAs enter the housing market by posting contracts; in the second stage, sellers sort themselves by

selecting a contract/REA, and buyers enter submarkets which are identifiable by the supply of real estate services, a . When buyers match with sellers, they implement competitive bidding strategies to purchase the house. Equilibria are constructed by solving backward. Equilibrium search strategies take as given the set of real estate contracts. This pins down the arrival rate of buyers and the expected number of sellers of each type attracted to a particular contract. In the first stage, REAs correctly anticipate the search behaviour of buyers and sellers in the second stage subgame. Taking as given the contracts posted by other agents, a REA enters the market and offers contract (a, z) if it is profitable to do so. An important aspect of REAs in the model is that the services they provide are perfectly observable to all buyers. This is an appropriate assumption for marketing and advertising services: yard signs, billboards, newspaper advertisements, and online listings are all designed to be seen, and buyers recognize the difference between, for example, an MLS listing and a for-sale-by-owner listing. High-end services, such as home staging, professional photographs of the dwelling, and extravagant open house events do not go unnoticed.

Adding REAs to the model in this manner introduces several more layers of analytical complexity. The essential intuition, however, obtains even when the services provided by REAs are completely valueless but still observable to other market participants. A straightforward way to impose such an environment is to set $\kappa(a) = \kappa_0$ for all a . Increasing a has no direct benefit to a potential buyer or the seller, but with a observable it becomes feasible for sellers to spend resources on REAs as a means of signalling their type. I proceed by investigating when even ineffective REAs play a role in the housing market. In the version of the model with $\kappa'(a) < 0$, the direct benefit of increasing a can reinforce an endogenous sorting mechanism, but the analytical results derived in the simpler environment illuminate the main workings of the model.

3.1 Real Estate Agents in an Environment with $\kappa(a) = \kappa_0$

Under the assumption that $\kappa(a) = \kappa_0$ for all a , the level of real estate services has no economic interpretation except that it can act as an observable market signal and affect beliefs about the buyer-seller ratio, θ , and the composition of sellers, σ . In this environment, market separation may seem unlikely without a sorting condition imposed on preferences or technologies. Despite the lack of an exogenously imposed single crossing property, sorting can arise endogenously when the real estate market is characterized by a directed search framework. REAs post contracts, effectively creating submarkets that can be distinguished by the observable real estate services, a . Buyers and sellers then direct their search to the different submarkets.

Perfect competition and free entry in the market for REAs ensure that commission fees will be bid down to earn zero profit.¹⁰ Let $\mathbb{C}_0$ denote the set of zero profit contracts:

$$\mathbb{C}_0 = \{(a, z) | a \geq 0, z = \phi(a)\} \quad (8)$$

The precise set of contracts offered by REAs and accepted by sellers is determined endogenously in a directed search equilibrium. What follows is a formal definition of the equilibrium search strategies of buyers and sellers, taking as given a set of real estate contracts, $\mathbb{C}_P \subset \mathbb{C}_0$, with observable component $\mathbb{M}_P = \text{proj}_1(\mathbb{C}_P)$, where proj_i is the projection map on the i th coordinate. Definition 3.1 takes into account the optimal bidding strategies of buyers and the optimal accept/reject decisions of sellers and focuses instead on equilibrium search behaviour. Next, a definition of an equilibrium with REA entry determines the set of equilibrium contracts, $\mathbb{C}_P$.

Definition 3.1. Given a set of real estate contracts $\mathbb{C}_P \subset \mathbb{C}_0$ with observable component

¹⁰To prove this claim, suppose instead that some REA offering (a, z) earns positive profit. Free entry of REAs implies that a new REA can offer $(a, z - \varepsilon)$ with $\varepsilon > 0$. With perfect competition in the market for REAs, every seller in submarket a will then choose the new contract over the original one. Moreover, since a is unchanged buyers' beliefs about seller types and submarket tightness remain the same. Finally, since ε can be arbitrarily small, it can be chosen so that the new real estate agent earns positive profit. REA entry remains profitable until $z = \phi(a)$.

$\mathbb{M}_P = \text{proj}_1(\mathbb{C}_P)$, equilibrium search strategies are characterized by a distribution of buyers Γ on $\mathbb{C}_P$ with support $\mathbb{C}_P^*$, a function for the buyer-seller ratio, $\theta : \mathbb{M}_P \rightarrow [0, \infty]$, a function for the composition of sellers, $\sigma : \mathbb{M}_P \rightarrow [0, 1]$, and a pair of seller values $\{\bar{V}_A, \bar{V}_R\}$ satisfying the following:

1. Buyers' optimal entry:

$$U(\sigma(a), \theta(a)) \leq \kappa_0 \text{ for all } (a, z) \in \mathbb{C}_P \quad (\text{with equality if } (a, z) \in \mathbb{C}_P^*).$$

2. Sellers' optimal sorting:¹¹ Define $\bar{V}_A = \bar{V}_R = 0$ if $\mathbb{C}_P^* = \emptyset$, otherwise

$$\bar{V}_A = \max \left\{ 0, \max_{(a,z) \in \mathbb{C}_P^*} V_A(z, \sigma(a), \theta(a)) \right\} \quad \text{and} \quad \bar{V}_R = \max \left\{ 0, \max_{(a,z) \in \mathbb{C}_P^*} V_R(z, \sigma(a), \theta(a)) \right\}.$$

- (i) For any $(a, z) \in \mathbb{C}_P$, $V_A(z, \sigma(a), \theta(a)) \leq \bar{V}_A$ (with equality if $(a, z) \in \mathbb{C}_P^*$ and $\sigma(a) > 0$). If $V_A(z, \sigma(a), \theta(a)) < \bar{V}_A$ and $\sigma(a) > 0$, then $\theta(a) = \infty$.
- (ii) For any $(a, z) \in \mathbb{C}_P$, $V_R(z, \sigma(a), \theta(a)) \leq \bar{V}_R$ (with equality if $(a, z) \in \mathbb{C}_P^*$ and $\sigma(a) < 1$). If $V_R(z, \sigma(a), \theta(a)) < \bar{V}_R$ and $\sigma(a) < 1$, then $\theta(a) = \infty$.

3. Market clearing:

$$\int_{\mathbb{C}_P^*} \frac{\sigma(a)}{\theta(a)} d\Gamma(a, z) \leq \sigma_0 S \quad (\text{with equality if } \bar{V}_A > 0),$$

and

$$\int_{\mathbb{C}_P^*} \frac{1 - \sigma(a)}{\theta(a)} d\Gamma(a, z) \leq (1 - \sigma_0) S \quad (\text{with equality if } \bar{V}_R > 0).$$

The first two parts of Definition 3.1 specify optimal entry into submarkets on the part of buyers and sellers. For instance, 2(i) requires that anxious sellers do not enter a submarket unless it enables them to achieve their highest possible payoff. Part (ii) is the analogous requirement for type R sellers. The final part of Definition 3.1 ensures that every seller

¹¹Sellers' payoffs, denoted $V_A(z, \sigma, \theta)$ and $V_R(z, \sigma, \theta)$, are defined as in Section 2.2 net of REA fees.

enters a submarket, but only if sellers find it worthwhile to list their house for sale in at least one of the existing submarkets.

What is missing from Definition 3.1 is the equilibrium behaviour of REAs. While Definition 3.1 restricts attention to REA contracts that earn zero profit, further refinements are needed to fully endogenize the set of equilibrium contracts. REAs play a market-making role, creating submarkets by constructing new listing agreements. An equilibrium set of contracts must be such that no other contract can be introduced to earn positive profit. This restriction requires specifying the beliefs about submarket tightness, θ , the composition of sellers, σ , and the commission fee, z , for real estate contracts that are not offered in equilibrium. A directed search equilibrium is such that no REA can offer an out-of-equilibrium listing contract and earn positive profit given the correctly anticipated equilibrium search strategies of buyers and sellers. An equivalent characterization of a directed search equilibrium rules out a candidate set of contracts $\mathbb{C}_P$ if there exists a zero profit deviation that can improve the expected payoffs to sellers participating in the new submarket.¹²

Definition 3.2. A directed search equilibrium in the housing market with REAs is a set of real estate contracts $\mathbb{C}_P$ with $(0, 0) \in \mathbb{C}_P$, a distribution of buyers Γ on $\mathbb{C}_0$ with support $\mathbb{C}_P$, functions $\theta : \mathbb{M}_0 \rightarrow [0, \infty]$ and $\sigma : \mathbb{M}_0 \rightarrow [0, 1]$, and values $\{\bar{V}_A, \bar{V}_R\}$ satisfying the following:

1. REAs offer zero profit contracts: $\mathbb{C}_P \subset \mathbb{C}_0$; and $\{\Gamma, \theta, \sigma, \bar{V}_A, \bar{V}_R\}$ satisfy Definition 3.1 given the set of contracts $\mathbb{C}_P$.
2. For any $(a', z') \in \mathbb{C}_0 \setminus \mathbb{C}_P$,

$$V_A(z', \sigma(a'), \theta(a')) \leq \bar{V}_A \quad \text{and} \quad V_R(z', \sigma(a'), \theta(a')) \leq \bar{V}_R$$

¹²Equivalence follows from the following argument: a listing contract that attracts some sellers and makes them strictly better off can be restructured to divide the extra surplus between the seller and the agent. Inversely, if a profitable deviation is possible, the real estate agent could instead pass some of the surplus on to his clients. This equivalent characterization is applied here in order to avoid introducing extra notation for beliefs regarding submarkets with real estate contracts that earn strictly positive profit. The assumption is maintained that upon observing a , a prospective buyer deduces that the commission charged to the seller is $z = \phi(a)$.

where $\{\Gamma, \theta, \sigma, \bar{V}_A, \bar{V}_R\}$ satisfy Definition 3.1 given $\mathbb{C}_P \cup (a', q')$.

First note that $(0, 0) \in \mathbb{C}_P$, which means that sellers maintain the option not to hire a REA: the for-sale-by-owner option. Part 1 of the definition then states that entry and search behaviour of buyers and sellers are optimal given the posted set of zero profit listing agreements. Part 2 states that no out-of-equilibrium contract can benefit sellers. This requires the beliefs about θ and σ for out-of-equilibrium submarkets to be consistent with the search behaviour of buyers and sellers in the subgame that includes the additional deviation under consideration. The resulting buyer-seller ratio, $\theta(a')$, has to be consistent with the free entry condition for buyers. Similarly, the resulting composition of sellers, $\sigma(a')$, must reflect the equilibrium search strategies of sellers following the introduction of contract $(a', z') \in \mathbb{C}_0 \setminus \mathbb{C}_P$.

The endogenous distribution of sellers and buyer-seller ratios across submarkets can induce a local single-crossing property and initiate sorting. Before formally characterizing the directed search equilibrium, an informal argument is given to suggest that it can be self-fulfilling to believe that anxious sellers separate themselves by hiring REAs. Suppose $\sigma_0 > (v - c_R)/(v - c_A)$ and consider a pooling equilibrium without REAs as in Section 2.4. A REA enters the market and decides to offer a listing agreement (a, z) with zero profit commission $z = \phi(a) > 0$. If buyers anticipate a higher share of anxious sellers among those represented by the REA, $\sigma(a) > \sigma_0 > (v - c_R)/(v - c_A)$, then the free entry conditions imply a higher buyer-seller ratio. The payoff functions for sellers in the new submarket are

$$V_R(\phi(a), \sigma(a), \theta(a)) = [1 - (1 + \theta(a))e^{-\theta(a)}](v - c_R) - \phi(a) \quad (9)$$

$$V_A(\phi(a), \sigma(a), \theta(a)) = [1 - (1 + \theta(a))e^{-\theta(a)}](v - c_A) - \phi(a) \quad (10)$$

Differentiating these payoff functions with respect to θ yields

$$\frac{dV_R}{d\theta} = \theta e^{-\theta}(v - c_R) < \theta e^{-\theta}(v - c_A) = \frac{dV_A}{d\theta} \quad (11)$$

The benefit of additional buyers is higher for anxious sellers than for relaxed sellers. Hence, the advantage of signalling is higher for anxious sellers, while the cost affects them symmetrically. The endogenously determined composition of sellers and arrival rate of buyers generate a single crossing property; conceptually, if $\sigma(a)$ and $\theta(a)$ can adjust as buyers and sellers sort among submarkets until relaxed sellers are indifferent between the two markets, then anxious sellers will strictly prefer to hire the REA.

The piecewise nature of the payoff function for type A sellers in (3) introduces a complication. If $\sigma_0 \leq (v - c_R)/(v - c_A)$, type A sellers receive a positive payoff even in a bilateral match because buyers are making cautious take-it-or-leave-it offers to ensure the purchase of a home regardless of the seller's motivation. Although V_A is increasing in a when $\sigma(a) \in ((v - c_R)/(v - c_A), 1)$, anxious sellers might still prefer the original pooling submarket because of the bilateral bonus. Even if $\sigma_0 > (v - c_R)/(v - c_A)$, a fully separating equilibrium might not be feasible because anxious sellers can deviate to the market dominated by relaxed sellers and capture $c_R - c_A$ in a bilateral match. Market separation is only incentive compatible if the net benefit from signalling (increased liquidity¹³ less real estate fees) exceeds the opportunity cost of the informational rent (the bilateral bonus of $c_R - c_A$) captured in the submarket without REAs. The first effect dominates whenever the buyer-seller ratios are sufficiently low (i.e., if housing is sufficiently illiquid) that the benefit from an increase in market tightness, θ , is large. When demand is too high, the benefit of further increasing market tightness is insufficient to offset the appeal of the bilateral bonus in the type R market. The parameter most directly (but inversely) related to market tightness is κ_0 , the entry cost for buyers. When κ_0 is high, buyers are scarce and the potential benefit from signalling a high motivation to sell is sizeable. I proceed by characterizing the housing market equilibrium in terms of the parameters κ_0 and σ_0 .

Lemma 1. $\bar{V}_R = V_R(0, \sigma(0), \theta(0))$.

¹³A higher buyer-seller ratio improves the probability of a sale and the likelihood of a multilateral match with payoff $v - c_A$.

Lemma 2. *In a separating equilibrium, $\bar{V}_A = V_A(z_A, 1, \theta(a_A))$ for the zero profit real estate contract $(a_A, z_A = \phi(a_A))$ satisfying $V_R(z_A, 1, \theta(a_A)) = \bar{V}_R$.*

Lemma 3. *Market separation is incentive compatible with the pair of contracts $(a_R, z_R) = (0, 0)$ and (a_A, z_A) if and only if*

$$\kappa_0 \geq (v - c_A) \exp \left(\frac{c_A - c_R}{v - c_R} \right) \equiv \underline{\kappa} \quad (12)$$

Lemma 3 specifies the necessary and sufficient conditions for which the for-sale-by-owner option $(a_R, z_R) = (0, 0)$ and the listing contract (a_A, z_A) induce search behaviour by buyers and sellers according to Definition 3.1 that is consistent with full market separating. In keeping with the intuition above, condition (12) requires the market to be sufficiently thin (few buyers because of costly entry) for the liquidity effect to overwhelm the foregone bilateral bonus.

For market separation to satisfy the second condition in the definition of a directed search equilibrium with REAs, further parameter restrictions are required to ensure that no other real estate contract could be introduced to improve sellers' expected payoffs. One deviation of potential interest is a full pooling contract. The following Lemma characterizes the conditions necessary and sufficient for sellers to prefer a full pooling submarket over the pair of fully separating submarkets.

Lemma 4. *A full pooling contract $(a_0, z_0) \rightarrow (0, 0)$ dominates the pair of separating contracts (strict for at least one type) if and only if $\kappa_0 \in \mathcal{K}(\sigma_0)$, where*

$$\mathcal{K}(\sigma_0) = \begin{cases} [0, \underline{\kappa}) & \text{if } 0 < \sigma_0 \leq \frac{v - c_R}{v - c_A} \\ \emptyset & \text{if } \frac{v - c_R}{v - c_A} < \sigma_0 < \frac{v - c_A}{v - 2c_A + c_R} \\ [0, \bar{\kappa}(\sigma_0)] & \text{if } \frac{v - c_A}{v - 2c_A + c_R} \leq \sigma_0 < 1 \end{cases} \quad (13)$$

and

$$\bar{\kappa}(\sigma_0) \equiv \exp \left(\frac{(v - c_A)[1 + \log(\sigma_0(v - c_A))] - \sigma_0(v - c_A)[1 + \log(v - c_R)] - \sigma_0(c_R - c_A)[1 + \log(v - c_A)]}{(1 - \sigma_0)(v - c_A) - \sigma_0(c_R - c_A)} \right) \quad (14)$$

While (14) is difficult to interpret, it is in fact the indifference condition for type A sellers between pooling and separation, after imposing the binding incentive compatibility constraint for type R sellers and the buyers' free entry conditions. When $\kappa_0 \geq \underline{\kappa}$, the single crossing property precludes a pooling equilibrium. If the conditions of Lemma 4 are satisfied, a pooling contract can nonetheless be welfare improving. This leads to the typical equilibrium non-existence problem as in Rothschild and Stiglitz (1976). Lemmas 1, 2, 3 and 4 combine to form the necessary and sufficient conditions for a fully separating equilibrium in the housing market with REAs, which are stated in the following Proposition.

Proposition 3.1. *The pair of contracts, $(a_R, z_R) = (0, 0)$ and (a_A, z_A) , constitute a fully separating equilibrium if and only if $\kappa_0 \geq \underline{\kappa}$ and $\kappa_0 \notin \mathcal{K}(\sigma_0)$.*

Proposition 3.1 is consistent with the intuition developed earlier. The ratio of buyers to sellers in the housing market must be low in order for anxious sellers to engage in costly signalling by accepting real estate agreements with positive commission fees. When the entry cost κ_0 is low, the buyer-seller ratios are sufficiently high that the benefit of signalling is not enough to provide anxious sellers with the incentive to abandon the bilateral bonus. Proposition 3.1 also points to a relationship between the aggregate composition of sellers, σ_0 , and the existence of a fully separating equilibrium. When most sellers are anxious to sell, the full pooling submarket closely resembles the separating type A submarket: market tightness is high, and buyers make low offers of c_A in the event of a bilateral match. Therefore, as the population of sellers becomes relatively homogeneous (i.e., as $\sigma_0 \rightarrow 1$), paying agency fees to achieve full market segmentation becomes unjustifiable.

Proposition 3.1 is reminiscent of the endogenous market segmentation result in Fang (2001). In Fang's paper, social culture is a seemingly irrelevant activity that can be used as an endogenous signalling device to partially overcome an information problem in the labour market. Here, if the parameters are conducive to separation, the hiring of irrelevant but costly real estate agents is used to signal type. Buyers form different beliefs about the composition of sellers in each separate submarket. Given these beliefs, anxious and relaxed sellers face

different incentives to join a particular submarket. The advantage of listing a house with a costly REA is a higher arrival rate of buyers, which results in a higher probability of trade. Because sellers differ in their reservation values, (a_A, z_A) can be carefully chosen by REAs so that relaxed sellers are just indifferent between the two submarkets, while anxious sellers strictly prefer the one with REAs. Embedding an endogenous market segmentation result in a directed search framework with profit maximizing market makers thus rules out Pareto inferior signalling equilibria.

It is of interest to study housing market equilibria that involve pooling. For instance, under what parameter restrictions are there partial or full pooling equilibria? Lemma 5 and Proposition 3.2 fill in these details, and Figure 2 provides a graphical representation.

Lemma 5. *Any hybrid equilibrium in which sellers of type $i \in \{A, R\}$ participate in both brokered and non-brokered submarkets, if it exists, is payoff equivalent to a fully separating equilibrium.*

The proof of Lemma 5 reveals that partial pooling violates the conditions of Definitions 3.1 and 3.2 in most situations. For certain parameter combinations the incentive compatibility constraints simultaneously bind for both types. In such cases, there could be multiple equilibria that are payoff equivalent, one of which involves full market separation.

Proposition 3.2. *Suppose $\kappa_0 < \underline{\kappa}$ or $\kappa_0 \in \mathcal{K}(\sigma_0)$. Then,*

1. *if $\sigma_0 > \frac{v-c_R}{v-c_A}$, the model has no equilibrium; and*
2. *if $\sigma_0 \leq \frac{v-c_R}{v-c_A}$, there exists a full pooling equilibrium.*

If $\sigma_0 > (v-c_R)/(v-c_A)$ and $\kappa_0 < \underline{\kappa}$, a pooling contract does not constitute an equilibrium because a deviating REA can offer a listing agreement with a positive commission to attract only the anxious sellers. Once the anxious sellers exit the pooling submarket, buyers alter their bidding strategy and offer c_R instead of c_A in a bilateral match. This change in buyers' behaviour affects the expected payoffs such that anxious sellers' search decision is no longer

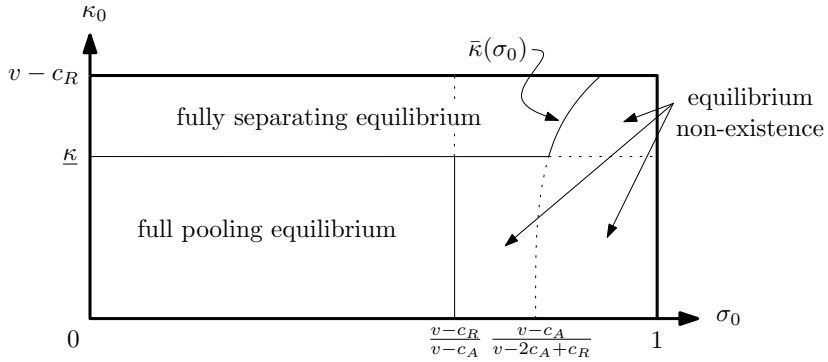


Figure 2: Graphical characterization of the housing market equilibrium with REAs.

optimal. This is the intuition behind the equilibrium non-existence problem in part 1 of Proposition 3.2. When $\sigma_0 \leq (v - c_R)/(v - c_A)$, the share of anxious sellers is low enough that buyers cautiously offer c_R in a bilateral match even in a pooling submarket in order to guarantee a successful home purchase. If $\kappa_0 < \bar{\kappa}$, there is no deviation that will attract only the motivated sellers.

3.2 Real Estate Agents in an Environment with $\kappa'(a) < 0$

The intuition developed in the previous section is still relevant when the economic importance of real estate services, a , is derived from the monotonic relationship with κ , the buyer's search cost. For notational convenience, the signalling role of real estate services a and the direct economic benefit of decreasing κ via a can be collapsed by considering contracts of the form (κ, z) . Let $\psi(\kappa)$ denote the implicit cost function REAs face when supplying the level of service required to reduce the search cost from κ_0 to κ .

When $\kappa'(a) < 0$, the effect of REA services on κ affects market tightness θ directly via the free entry condition, which then enters the sellers' payoff functions. Unlike in the environment with $\kappa'(a) = 0$, the REA technology directly imposes a local sorting condition.¹⁴ Sellers

¹⁴The sorting condition is local, and REA contracts do not necessarily lead to equilibrium market separation for the same reasons both pooling and separation are possible equilibrium outcomes in Section 3.1.

hiring real estate agents are effectively paying for higher matching probabilities by subsidizing the entry of buyers. Anxious sellers, by their very nature, benefit relatively more from the marginal reduction in κ . Consequently, anxious sellers should tolerate higher commission fees in equilibrium for two reasons: they have a higher willingness to pay REAs to increase θ directly by lowering buyers' search costs, and also indirectly by affecting buyers' beliefs about the composition of sellers. The fact that κ is a sorting variable therefore reinforces the signalling role of REAs. This environment is an appropriate fit for the North American housing markets if the marketing efforts of a realtor do not directly warrant compensation between 5 and 7 percent of the price. Hiring a full service agent can still be worthwhile for motivated sellers because listing the house on the Multiple Listing Service (MLS) signals a high willingness to sell and generates additional visits from potential buyers.

This provides a theoretical foundation for the empirical results of Hendel, Nevo, and Ortalo-Magné (2009). They compare housing market transactions on two different marketing platforms: the MLS and the newly established low cost FSBO Madison. They find that after controlling for observable house characteristics, the precommission sale prices are similar between the two platforms, but that homes listed with a traditional real estate broker have shorter times on the market and are more likely to ultimately result in a transaction. They also find evidence of endogenous sorting and report that impatient sellers are more likely to list with the high commission, high service option. These findings are consistent with the main theme of this paper. A higher buyer-seller ratio for houses listed on the MLS and the higher level of services provided by full-commission REAs lead to a higher probability of a sale.

Levitt and Syverson (2008a) similarly compare limited-service and full-service REAs. Time on the market is longer for houses sold with the assistance of less costly realtors, but sale prices are not significantly different. Bernheim and Meer (2008) study Stanford Housing listings and find that sellers realize similar prices but sell less quickly when they select not to hire an agent. These empirical observations and the predictions of the theory point to REAs and the MLS as primarily fulfilling a liquidity role in the housing market, rather than

directly affecting the expected sale price of a home.¹⁵

3.3 Constrained Efficiency with Real Estate Agents

With endogenous real estate services, the constrained planner chooses market tightness θ , and entry cost κ to maximize the (per seller) social surplus:

$$\max_{\theta, \kappa} [1 - e^{-\theta}] (v - c) - \theta \kappa - \psi(\kappa) \quad (15)$$

The first order conditions with respect to θ and κ are

$$e^{-\theta} (v - c) = \kappa \quad (16)$$

$$\theta = \psi'(\kappa) \quad (17)$$

The first condition pins down the optimal buyer-seller ratio by equating the cost of entering the market, κ , with the marginal social surplus of having an additional buyer searching for a house. The additional condition stemming from the optimal choice of κ equates the marginal benefit of real estate services for all buyers with the marginal cost to REAs, $\psi'(\kappa)$. The efficient allocation with heterogeneous sellers is simply the separating allocation described in Section 2.3 with the additional restriction that κ satisfy equation (17).

In a fully separating equilibrium with REAs, the type R submarket achieves the efficient level of real estate services and efficient buyer entry. The type A submarket, on the other hand, involves excess spending on REAs, which is the signalling cost required to induce

¹⁵While the empirical result that sales prices are indistinguishable between the two marketing platforms in Hendel, Nevo, and Ortalo-Magné (2009), Levitt and Syverson (2008a), and Bernheim and Meer (2008) contradicts the prediction of the model, a straightforward modification can realign the theory and evidence. In a previous version of this paper, Stacey (2012), REAs provide services that can improve the expected quality of a match. In other words, REAs offer marketing and home staging services that can increase a buyer's valuation of the home. Then, in a separating equilibrium, the lower take-it-or-leave-it offer in a type A bilateral match reduces the average sale price, but higher market tightness and superior marketing services have the opposite effect. Depending on the parameters of the model, expected precommission prices can be identical across the two submarkets despite distinct transaction price distributions.

efficient buyer entry. Full separation in an equilibrium with REAs is welfare improving, but does not achieve the solution to the constrained planner’s problem because of the inefficiencies required to make the type *A* real estate contract incentive compatible. These welfare results contrast those of Albrecht, Gautier, and Vroman (2010). In their paper, partial commitment to the list price yields an efficient separating equilibrium without the efficiency loss from costly signalling. By modelling the agreement between a seller and her agent, I have shown that while separation is possible under certain parameters, the first best allocation remains unattainable.

3.4 Fixed Rate Commissions

The analysis thus far deals with flat fee commissions charged by REAs. In practice, a fixed rate commission structure is more common: real estate contracts in North America typically specify a commission of 5 to 7 percent of the sale price (Hsieh and Moretti, 2003; Federal Trade Commission and U.S. Department of Justice, 2007). From a principal-agent perspective,¹⁶ a real estate fee that increases with the sale price is more likely to induce effort on the part of the real estate agent, whereas upfront non-refundable fees are least effective at motivating the agent. While I abstract from principal-agent matters in this paper, it is important to check the robustness of the results when listing contracts are modelled to reflect the type of contract commonly observed between a seller and her agent.

Restricting the analysis to fixed rate contracts introduces two additional effects that further hinder full market separation. First, when the commission is specified as a fraction of the sale price, a buyer has to increase his take-it-or-leave-it offer in a bilateral match until the seller deems it acceptable after real estate fees are deducted. More specifically,

¹⁶Many theoretical models of real estate agents focus on the principal-agent relationship between the seller (the principal) and the realtor (the agent) (Yavaş, 1992; Yavaş and Yang, 1995). The attention of empiricists has also been aimed at the principal-agent problem in the market for real estate services. Levitt and Syverson (2008b) and Rutherford, Springer, and Yavaş (2005) find evidence to support the hypothesis that sellers’ and their agents’ incentives are misaligned by comparing the selling prices and duration on the market in transactions when the real estate agent is a third party and when the agent is also the owner of the home.

when the commission is z percent and the seller is willing to accept c , the offer must be at least $c/(1 - z)$. This reduces the payoff to a buyer in a bilateral match and implies that buyer entry is affected by the commission rate. Higher fees result in fewer buyers, which offsets the desired effect of attracting more buyers with REAs. Second, fixed rate contracts affect the incentive for relaxed sellers to mimic because payment to REAs is contingent on a transaction. To see why this is important, compute the expected real estate fee to be paid by an anxious seller in a type A submarket with commission rate z_A :

$$\theta_A e^{-\theta_A} \frac{z_A c_A}{1 - z_A} + [1 - (1 + \theta_A) e^{-\theta_A}] z_A v \quad (18)$$

The first term is the commission paid on the take-it-or-leave-it offer of $c_A/(1 - z_A)$ in a bilateral match, and the second term is the commission paid when two or more buyers arrive. When a relaxed seller accepts the (κ_A, z_A) contract and lists her home in the type A submarket, the expected commission fee is only

$$[1 - (1 + \theta_A) e^{-\theta_A}] z_A v \quad (19)$$

An offer is rejected by a mimicker in a bilateral match since $c_R > c_A$, and the REA only collects the commission when two or more buyers visit a relaxed seller's house in the anxious sellers' submarket. Since both types of sellers receive zero payoff in a bilateral match, this does not affect the incentive compatibility constraint directly. Instead, the zero profit conditions in the market for REAs imply that mimicking sellers can essentially free ride on the commissions paid by anxious sellers. This makes the type A submarket more appealing relative to the type R submarket where relaxed sellers bear the full burden of marketing costs. The two effects just described work against incentive compatibility and full market separation. Nonetheless, the analysis is not fundamentally altered when fixed rate contracts are imposed: it merely implies that a smaller parameter space generates a fully separating equilibrium.

Listing agreements typically specify a list price. What if the REA's commission can be

made contingent on procuring a “ready, willing, and able” buyer (i.e., contingent on receiving an offer at or above the list price)? This form of contract is often observed in North American real estate markets.¹⁷ Even if the seller rejects an offer equal to or above the list price, it is considered that the REA has provided the agreed upon services and the seller must still pay the commission. The “ready, willing, and able” clause (hereinafter, the RWA clause) is useful for (but not essential for) generating a separating equilibrium.

Proposition 3.3. *Adding the list price and a RWA clause to the real estate contract tightens the incentive compatibility constraint for relaxed sellers.*

This structure of real estate contract dissuades patient sellers from mimicking impatient ones and entering the market with higher demand. The contract introduces a penalty for rejecting a take-it-or-leave-it offer in a bilateral match and it becomes less costly for anxious sellers to signal their type. Consider a listing agreement designed for type A sellers with the list price $p_A = c_A/(1 - z_A)$. Type A sellers are indifferent to the RWA clause because they are already willing to accept an offer of $c_A/(1 - z_A)$. Type R sellers, on the other hand, must now pay a cost in a bilateral match if they choose to list their house at p_A . The extra cost to mimickers makes it easier for REAs to offer incentive compatible contracts that separate sellers by type. If list prices are determined strategically, it is possible that an anxious seller’s expected payoff can be further enhanced. While a list price in $[c_A, c_A/(1 - z_A))$ does adversely affect even anxious sellers, it might sting less than the direct cost of the agency fee. In other words, there is the possibility that simultaneously lowering z_A and p_A improves the expected payoff to type A sellers without attracting type R sellers. Thus, RWA clauses effectively mitigate both of the unfavourable incentive effects associated with fixed rate commissions.

¹⁷For example, a listing agreement with the Toronto Real Estate Board stipulates that “the Seller agrees to pay the Listing Brokerage a commission of% of the sale price of the Property or for any valid offer to purchase the Property from any source whatsoever obtained during the Listing Period and on the terms and conditions set out in this Agreement.”

4 Concluding Remarks

In this paper I present a model of the market for housing using a search framework that captures the realistic and strategic interaction between buyers and sellers in determining transaction prices. The model reflects differences in sellers' willingness to make a sale. Private information about a seller's motivation leads to an inefficient equilibrium with illiquid real estate. Some buyer-seller matches fail to result in a transaction despite the positive gains from trade. Reduced entry of buyers can further impact the volume of trade in the housing market. By introducing real estate agents into the model, there is a potential for housing market segmentation to alleviate the information problem and improve efficiency.

The model can qualitatively account for many of the observed realtor facts in residential real estate markets. For instance, 88 percent of home sellers choose to enlist the services of a real estate agent according to a 2010 survey conducted by the National Association of Realtors. This percentage has remained high in recent years, despite evidence suggesting that the value of the services provided by real estate agents, measured in terms of transaction prices and time on the market, is not enough to justify high commission rates between five and seven percent. With seller heterogeneity and incomplete information, the theory sheds light on the demand for real estate services. Realtors not only provide valuable marketing/matching services, but can also structure their contracts in a way that offers a potential solution to the adverse selection problem. A motivated seller can select a particular listing agreement as a means of signalling a high willingness to sell, thus attracting more buyers. In fact, I show that the demand for agency can, in some circumstances, be maintained even when the services offered by agents provide no direct benefit to buyers or sellers.

The listing agreement between a seller and her real estate agent typically specifies the commission as a fixed percentage of the sale price, and outlines the broker's right to earn a commission for any valid offer to purchase the house. In the real estate agent literature, the listing contract is often studied from the perspective of a principal-agent relationship. Many have noted the apparent inefficiency in applying a fee structure that fails to completely

align the incentives of the seller and the agent. A better commission structure in a simple (linear) model of incentive compensation would require a lower intercept and a steeper slope. Nevertheless, real estate contracts with a constant percentage commission have persisted for decades. In this paper, I analyze the listing contract from a different perspective and offer an alternative explanation for the structure of real estate contracts. I show that the “ready, willing, and able” clause is related to (but not essential for) a mechanism that can induce sellers to truthfully reveal their willingness to sell. Interestingly, the clause would be less effective if the fee structure was altered according to the solution to an agency problem.

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A Omitted Proofs

Proof of Proposition 2.1. To show that buyer entry is optimal, denote by Π_A the social surplus from putting a house on the market when the seller has reservation value c_A . As long as one or more potential buyers show up, the surplus is $v - c_A$.

$$\Pi_A(\theta) = e^{-\theta} \sum_{k=1}^{\infty} \frac{\theta^k}{k!} (v - c_A) = (1 - e^{-\theta})(v - c_A) \quad (\text{A.1})$$

Define Π_R in the analogous manner for houses available for purchase from relaxed sellers. Taking the numbers of sellers as given, the planner has only to choose the number of buyers visiting sellers of each type to maximize total social surplus less entry costs. Equivalently, the social planner can choose θ_A and θ_R to maximize the average social surplus per house.

$$\max_{\theta_A, \theta_R} \sigma_0 [\Pi_A(\theta_A) - \kappa_0 \theta_A] + (1 - \sigma_0) [\Pi_R(\theta_R) - \kappa_0 \theta_R] \quad (\text{A.2})$$

After substituting for Π_A using the definition in equation (A.1) and likewise for Π_R , the first order conditions for the planner's problem are

$$e^{-\theta_A}(v - c_A) = \kappa_0 \quad (\text{A.3})$$

$$e^{-\theta_R}(v - c_R) = \kappa_0 \quad (\text{A.4})$$

These are the same equations as the free entry conditions for buyers in the full information benchmark housing market, equations (6) and (7). $\square$

Proof of Proposition 2.2. With list prices, the housing market can potentially be characterized by multiple submarkets. Consider one submarket for sellers with list prices p_1 , and another submarket for sellers with list price p_2 . Assume without loss of generality that $\sigma_1 \geq \sigma_2$.

$\sigma_1 \geq \sigma_2$ requires relaxed sellers to (weakly) prefer submarket 2: $V_R(\sigma_2, \theta_2) \geq V_R(\sigma_1, \theta_1)$.

Differentiating the payoff to a relaxed seller, $V_R(\sigma, \theta) = [1 - (1 + \theta)e^{-\theta}](v - c_R)$, yields

$$\frac{dV_R}{d\theta} = \theta e^{-\theta}(v - c_R) > 0 \quad (\text{A.5})$$

$V_R(\sigma_2, \theta_2) \geq V_R(\sigma_1, \theta_1)$ therefore requires $\theta_2 \geq \theta_1$. Relaxed sellers choose an appropriate list price to participate in the submarket with the highest buyer-seller ratio.

The buyer-seller ratios are determined by the free entry conditions, $U(\sigma, \theta) = \kappa_0$ for $\sigma \in \{\sigma_1, \sigma_2\}$, with

$$U(\sigma, \theta) = \begin{cases} e^{-\theta}\sigma(v - c_A) & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\ e^{-\theta}(v - c_R) & \text{if } \sigma \leq \frac{v - c_R}{v - c_A} \end{cases} \quad (\text{A.6})$$

Differentiating the free entry condition and rearranging yields

$$\frac{d\theta}{d\sigma} = \begin{cases} 1/\sigma & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\ 0 & \text{if } \sigma \leq \frac{v - c_R}{v - c_A} \end{cases} \quad (\text{A.7})$$

which implies $\theta_1 \geq \theta_2$ given $\sigma_1 \geq \sigma_2$.

Combining the optimal price posting decision of a relaxed seller (which requires $\theta_2 \geq \theta_1$) and the implication of the free entry conditions ($\theta_1 \geq \theta_2$) yields the result that $\theta_1 = \theta_2$. Buyer entry and bidding strategies are such that list prices are meaningless: the equilibrium is indistinguishable from random search. $\square$

Proof of Lemma 1. Suppose (for the sake of contradiction) that $V_R(0, \sigma(0), \theta(0)) < \bar{V}_R$, which implies either $\theta(0) = \infty$ or $\sigma(0) = 1$. In the first case,

$$V_R(0, \sigma(0), \theta(0)) = v - c_R \geq \bar{V}_R \quad (\text{A.8})$$

A contradiction. In the latter case, satisfying part 3 of Definition 3.1 requires some other submarket $(a, z) \in \mathbb{C}_P^*$ with $\sigma(a) < 1$ such that

$$\bar{V}_R = V_R(z, \sigma(a), \theta(a)) \quad (\text{A.9})$$

But $\sigma(a) < 1$ implies $\theta(a) < \theta(0)$ by part 1 of Definition 3.1. This along with $z \geq 0$ implies

$$V_R(z, \sigma(a), \theta(a)) < V_R(0, 1, \theta(0)) \quad (\text{A.10})$$

A contradiction. □

Proof of Lemma 2. Suppose (for the sake of contradiction) that the REA listing contract is $(a, z) \neq (a_A, z_A)$ in a separating equilibrium. Any $z < z_A$ would violate incentive compatibility. With $z > z_A$, the incentive compatibility constraint for relaxed sellers is slack:

$$\bar{V}_R = [1 - (1 + \theta(0))e^{-\theta(0)}] (v - c_R) > -z + [1 - (1 + \theta(a))e^{-\theta(a)}] (v - c_R). \quad (\text{A.11})$$

Consider a new zero profit arrangement (a', z') with $a' < a$ so that $z' = \phi(a') < \phi(a) = z$. With (A.11) not binding, $a' \in (a_A, a)$ would not attract relaxed sellers, and consequently $\sigma(a') = 1$ and $\theta(a') = \theta(a)$. Since the new arrangement is less costly in terms of REA fees, anxious sellers would achieve a higher expected payoff, which violates part 2 of Definition 3.2. □

Proof of Lemma 3. The relevant expected payoffs, assuming a separating equilibrium, are

$$V_A(0, 0, \theta_R) = [1 - (1 + \theta_R)e^{-\theta_R}](v - c_A) + \theta_R e^{-\theta_R}(c_R - c_A) \quad (\text{A.12})$$

$$V_R(0, 0, \theta_R) = [1 - (1 + \theta_R)e^{-\theta_R}](v - c_R) \quad (\text{A.13})$$

$$V_A(z_A, 1, \theta_A) = [1 - (1 + \theta_A)e^{-\theta_A}](v - c_A) - z_A \quad (\text{A.14})$$

$$V_R(z_A, 1, \theta_A) = [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - z_A \quad (\text{A.15})$$

The binding incentive compatibility constraint for relaxed sellers pins down the minimum fee, z_A :

$$\begin{aligned} V_R(0, 0, \theta_R) &= [1 - (1 + \theta_R)e^{-\theta_R}](v - c_R) \\ &= [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - z_A = V_R(z_A, 1, \theta_A) \end{aligned} \quad (\text{A.16})$$

Substituting the payoff functions from above, the incentive compatibility constraint can be rewritten

$$V_A(0, 0, \theta_R) - [1 - e^{-\theta_R}](c_R - c_A) = V_A(z_A, 1, \theta_A) - [1 - (1 + \theta_A)e^{-\theta_A}](c_R - c_A) \quad (\text{A.17})$$

The fully separating market arrangement is incentive compatible for anxious sellers if and only if $V_A(z_A, 1, \theta_A) \geq V_A(0, 0, \theta_R)$. Using the condition derived above, a separating equilibrium requires

$$[e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}](c_R - c_A) > 0 \quad \Longleftrightarrow \quad e^{\theta_A - \theta_R} \geq 1 + \theta_A \quad (\text{A.18})$$

The free entry conditions can be used to solve for θ_A and θ_R .

$$U(0, \theta_R) = e^{-\theta_R}(v - c_R) = \kappa_0 \quad \Longleftrightarrow \quad \theta_R = \log \left(\frac{v - c_R}{\kappa_0} \right) \quad (\text{A.19})$$

$$U(1, \theta_A) = e^{-\theta_A}(v - c_A) = \kappa_0 \quad \Longleftrightarrow \quad \theta_A = \log \left(\frac{v - c_A}{\kappa_0} \right) \quad (\text{A.20})$$

With these expressions for θ_A and θ_R , the inequality in (A.18) reduces to condition (12). $\square$

Proof of Lemma 4. First consider the case where $\sigma_0 \leq (v - c_R)/(v - c_A) \equiv \underline{\sigma}$. Since buyer entry and bidding strategies are identical in both the type R submarket and the pooling submarket, relaxed sellers are indifferent between the two: $V_R(0, 0, \theta_R) = V_R(0, \sigma_0, \theta_0)$. In contrast, a full pooling contract can only benefit anxious sellers if $V_A(0, \sigma_0, \theta_0) = V_A(0, 0, \theta_R) > V_A(z_A, 1, \theta_A)$. By Lemma 3, this is the case when $\kappa_0 < \underline{\kappa}$.

Next consider the case where $\sigma_0 > (v - c_R)/(v - c_A) \equiv \underline{\sigma}$. The relevant expected payoffs, assuming a separating equilibrium, are (A.12), (A.13), (A.14), (A.15), and

$$V_A(0, \sigma_0, \theta_0) = [1 - (1 + \theta_0)e^{-\theta_0}](v - c_A) \quad (\text{A.21})$$

$$V_R(0, \sigma_0, \theta_0) = [1 - (1 + \theta_0)e^{-\theta_0}](v - c_R) \quad (\text{A.22})$$

With $\sigma_0 > \underline{\sigma}$, relaxed sellers strictly prefer the pooling submarket because of the higher buyer-seller ratio implied by the free entry of buyers: $V_R(0, \sigma_0, \theta_0) > V_R(0, 0, \theta_R)$. Whether anxious sellers prefer the pooling submarket depends on κ_0 and σ_0 .

The incentive compatibility constraint for relaxed sellers, equation (A.16), pins down the optimal commission rate for full separation, z_A . After substituting the pooling payoff from (A.22), the constraint becomes

$$\begin{aligned} V_R(0, \sigma_0, \theta_0) + [(1 + \theta_0)e^{-\theta_0} - (1 + \theta_R)e^{-\theta_R}](v - c_R) \\ = [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - z_A \end{aligned} \quad (\text{A.23})$$

By substituting the type A payoff functions from (A.14) and (A.21), the constraint can be rewritten and rearranged to obtain

$$\begin{aligned} V_A(0, \sigma_0, \theta_0) - V_A(z_A, 1, \theta_A) = [(1 + \theta_R)e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}](v - c_R) \\ + [(1 + \theta_A)e^{-\theta_A} - (1 + \theta_0)e^{-\theta_0}](v - c_A) \end{aligned} \quad (\text{A.24})$$

The preference for pooling among anxious sellers, $V_A(0, \sigma_0, \theta_0) \geq V_A(z_A, 1, \theta_A)$, therefore requires

$$\frac{v - c_R}{v - c_A} \geq \frac{(1 + \theta_0)e^{-\theta_0} - (1 + \theta_A)e^{-\theta_A}}{(1 + \theta_R)e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}} \quad (\text{A.25})$$

Using the free entry conditions, we can substitute for the buyer-seller ratios to obtain

$$1 + \log\left(\frac{v - c_R}{\kappa_0}\right) - \left[1 + \log\left(\frac{v - c_A}{\kappa_0}\right)\right] \frac{v - c_R}{v - c_A} \geq \left[1 + \log\left(\frac{\sigma_0(v - c_A)}{\kappa_0}\right)\right] \frac{1}{\sigma_0} - \left[1 + \log\left(\frac{v - c_A}{\kappa_0}\right)\right] \quad (\text{A.26})$$

Rearranging to isolate κ_0 , this inequality reduces to

$$\begin{aligned} \kappa_0 &\geq \bar{\kappa}(\sigma_0) \quad \text{if } \underline{\sigma} < \sigma_0 < \bar{\sigma} \\ \kappa_0 &\leq \bar{\kappa}(\sigma_0) \quad \text{if } \bar{\sigma} \leq \sigma_0 < 1 \end{aligned} \quad (\text{A.27})$$

where $\bar{\sigma} \equiv (v - c_A)/(v - 2c_A + c_R)$ and

$$\bar{\kappa}(\sigma_0) \equiv \exp \left(1 + \frac{(v - c_A)[\log(\sigma_0(v - c_A)) - \sigma_0 \log(v - c_R)] - \sigma_0(c_R - c_A) \log(v - c_A)}{(1 - \sigma_0)(v - c_A) - \sigma_0(c_R - c_A)} \right) \quad (\text{A.28})$$

When $\sigma_0 \in (\underline{\sigma}, \bar{\sigma})$, the inequality can never be satisfied because it would imply an entry cost that prohibits buyer entry: $\sigma_0 \in (\underline{\sigma}, \bar{\sigma})$ and $\kappa_0 \geq \bar{\kappa}(\sigma_0)$ imply $\kappa_0 > \sigma_0(v - c_A)$.¹⁸ A pooling contract therefore cannot improve the expected payoff to anxious sellers when $\sigma_0 \in (\underline{\sigma}, \bar{\sigma})$. When $\sigma_0 \in [\bar{\sigma}, 1)$, on the other hand, model parameters that satisfy $\kappa_0 \leq \bar{\kappa}(\sigma_0)$ ensure that a pooling contract is preferred to the pair of separating contracts for both types of sellers (strict for the relaxed sellers). $\square$

Proof of Proposition 3.1. By Lemma 3, the pair of contracts $(a_R, z_R) = (0, 0)$ and (a_A, z_A) is incentive compatible when $\kappa_0 \geq \underline{\kappa}$. Lemmas 1 and 2 rule out other REA listing agreements that attract only one type of seller.

Suppose (for the sake of contradiction) that there is a deviation, $(a', z') \in \mathbb{C}_0$, that attracts both types of sellers and violates part 2 of Definition 3.2. Suppose first that the deviation does not attract all of the relaxed sellers and hence the incentive compatibility constraint for relaxed sellers binds:

$$V_R(z', \sigma(a'), \theta(a')) = \underbrace{V_R(0, 0, \theta_R)}_{=\bar{V}_R} = V_R(z_A, 1, \theta_A) \quad (\text{A.29})$$

¹⁸With $\sigma_0 \in (\underline{\sigma}, \bar{\sigma})$, the condition that $\bar{\kappa} > \sigma_0(v - c_A)$ can be written

$$\sigma_0[(1 + x)[1 - \log \sigma_0] + \log(1 - x)] < 1, \quad \sigma_0 \in (\underline{\sigma}, \bar{\sigma})$$

where $x \equiv (c_R - c_A)/(v - c_A) \in (0, 1)$. It is straightforward to show using Bernoulli's inequality that

$$\max_{\sigma \in (\underline{\sigma}, \bar{\sigma})} \{ \sigma[(1 + x)[1 - \log \sigma] + \log(1 - x)] \} = (1 + x)(1 - x)^{\frac{1}{1+x}} < 1.$$

In order for the introduction of (a', z') to violate part 2 of Definition 3.2, it must be that

$$V_A(z', \sigma(a'), \theta(a')) > \underbrace{V_A(z_A, 1, \theta_A)}_{=\bar{V}_A} \geq V_A(0, 0, \theta_R) \quad (\text{A.30})$$

If $\sigma(a') \in (0, \underline{\sigma}]$, then $\theta(a') = \theta_R$, which violates (A.30) since $z' \geq 0$. If instead $\sigma(a') \in (\underline{\sigma}, 1)$, then (A.29) and (A.30) imply

$$-z' + \left[1 - (1 + \theta(a'))e^{-\theta(a')}\right] (v - c_R) = -z_A + \left[1 - (1 + \theta_A)e^{-\theta_A}\right] (v - c_R) = \bar{V}_R \quad (\text{A.31})$$

and

$$-z' + \left[1 - (1 + \theta(a'))e^{-\theta(a')}\right] (v - c_A) > -z_A + \left[1 - (1 + \theta_A)e^{-\theta_A}\right] (v - c_A) = \bar{V}_A \quad (\text{A.32})$$

When $\theta(a')$ and θ_A are determined by the free entry conditions in part 1 of Definition 3.1, satisfying both (A.31) and (A.32) is not possible with $c_A < c_R$. This proves that the deviation (a', z') must result in an incentive compatibility constraint for relaxed sellers that is slack, and the deviation must therefore attract all relaxed sellers.

Suppose now that the deviation attracts only some of the anxious sellers. If $\sigma(a') \in (0, \underline{\sigma}]$, then $\theta(a') = \theta_R$ and $V_A(z', \sigma(a'), \theta(a')) \leq V_A(0, 0, \theta_R) \leq \bar{V}_A$. The other possibility is $\sigma(a') \in (\underline{\sigma}, \sigma_0)$ if $\underline{\sigma} < \sigma_0$. But since $\theta(a')$ is increasing in σ when $\sigma > \underline{\sigma}$, anxious sellers could achieve an even higher expected payoff from a deviation that attracts all sellers. It is therefore sufficient to consider only deviations that result in full pooling. However, $\kappa_0 \geq \underline{\kappa}$ and $\kappa_0 \notin \mathcal{K}(\sigma_0)$ exclude the parameter space where, according to Lemma 4, the introduction of a pooling contract would violate part 2 of Definition 3.2. $\square$

Proof of Lemma 5. Let $(z_N, \sigma_N, \theta_N) = (0, \sigma(0), \theta(0))$ describe the non-brokered submarket (i.e., the for-sale-by-owner (FSBO) market), and $(z_B, \sigma_B, \theta_B) = (\phi(a_B), \sigma(a_B), \theta(a_B))$ denote the brokered submarket with listing agreement (a_B, z_B) .

Consider pooling in the FSBO market. The following inequalities must hold, with at

least one of them binding:

$$\bar{V}_R \equiv [1 - (1 + \theta_N)e^{-\theta_N}] (v - c_R) \geq -z_B + [1 - (1 + \theta_B)e^{-\theta_B}] (v - c_R) \quad (\text{A.33})$$

and

$$\begin{aligned} \bar{V}_A &\equiv [1 - (1 + \theta_N)e^{-\theta_N}] (v - c_A) + \begin{cases} 0 & \text{if } \sigma_N > \frac{v - c_R}{v - c_A} \\ \theta_N e^{-\theta_N} (c_R - c_A) & \text{if } \sigma_N \leq \frac{v - c_R}{v - c_A} \end{cases} \\ &\geq -z_B + [1 - (1 + \theta_B)e^{-\theta_B}] (v - c_A) + \begin{cases} 0 & \text{if } \sigma_B > \frac{v - c_R}{v - c_A} \\ \theta_B e^{-\theta_B} (c_R - c_A) & \text{if } \sigma_B \leq \frac{v - c_R}{v - c_A} \end{cases} \end{aligned} \quad (\text{A.34})$$

If (A.34) holds with equality but (A.33) does not, then $\sigma_B = 1$. Consider a new zero profit arrangement (a', z') with $a' < a_B$ so that $z' = \phi(a') < \phi(a_B) = z_B$. Since (A.33) is not binding, a' sufficiently close to a_B would not attract relaxed sellers, and consequently $\sigma(a') = \sigma_B = 1$ and $\theta(a') = \theta_B$. The new arrangement is less costly in terms of REA fees, and hence anxious sellers would achieve an expected payoff greater than $\bar{V}_A$. This violates part 2 of Definition 3.2.

If (A.33) holds with equality but (A.34) does not, then $\sigma_B = 0$. For (A.33) to bind, it must be that $\theta_B > \theta_N$. To be consistent with the free entry conditions in part 1 of Definition 3.1, a necessary condition for $\theta_B > \theta_N$ is $\sigma_B > \sigma_N$, which in this case means $\sigma_N < 0$: a contradiction.

Finally, suppose both (A.33) and (A.34) hold with equality. For (A.33) to bind, it must be that $\theta_B > \theta_N$ and therefore $\sigma_B > (v - c_R)/(v - c_A)$. Given this, (A.34) can only hold with equality if $\sigma_N \leq (v - c_R)/(v - c_A)$. If $\sigma_B \in ((v - c_R)/(v - c_A), 1)$, a REA could offer (a', z') with $a' > a_B$ so that $z' = \phi(a') > \phi(a_B) = z_B$. With a' sufficiently close to a_B , indifference among relaxed sellers would imply $\sigma(a') > \sigma_B$ and hence $\theta(a') > \theta_B$. Since $v - c_A > v - c_R$, anxious sellers would strictly prefer the new arrangement, violating part 2 of Definition 3.2. If $\sigma_B = 1$, there is no such deviation that can attract only anxious sellers when both (A.33) and (A.34) hold with equality. Notice, however, that $\sigma_B = 1$ and $\sigma_N < (v - c_R)/(v - c_A)$, so

$\bar{V}_R$ and $\bar{V}_A$ are the same as they would be with full separation.

The arguments for the proof are similar when considering pooling in the brokered submarket. $\square$

Proof of Proposition 3.2. When $\kappa_0 < \underline{\kappa}$ or $\kappa_0 \in \mathcal{K}(\sigma_0)$, a fully separating equilibrium cannot be achieved (by Lemmas 3 and 4). By Lemmas 5 and 1, this leaves only the possibility of a full pooling equilibrium without REAs: $\sigma(0) = \sigma_0$ and $\theta(0) = \theta_0$.

Proof of part 1: $\sigma_0 > (v - c_R)/(v - c_A) \equiv \underline{\sigma}$ implies $\theta_0 = \log(\sigma_0(v - c_A)/\kappa)$ and the absence of a bilateral bonus. Since $\theta(a)$ is increasing in $\sigma(a) \in (\underline{\sigma}, 1)$ by the free entry conditions in part 1 of Definition 3.1, an incentive compatible zero profit contract with $a > 0$ can be designed to attract the type A sellers without attracting the type R sellers. This precludes the existence of a full pooling equilibrium when $\sigma_0 > \underline{\sigma}$.

Proof of part 2: Take as given a full pooling submarket with $\sigma_0 \leq \underline{\sigma}$ and consider a REA contract (a, z) with $a > 0$. If $\sigma(a) \leq \underline{\sigma}$, the new submarket does not attract any sellers since $\theta(a) = \theta_0$ and $z > 0$. If $\sigma(a) > \underline{\sigma}$, it must be the case that the relaxed sellers achieve at most $\bar{V}_R \equiv [1 - (1 + \theta_0)e^{-\theta_0}](v - c_R)$ in the new submarket (otherwise all relaxed sellers would list with the REA and we would have $\sigma(a) \leq \sigma_0 \leq \underline{\sigma}$). Therefore,

$$z \geq [(1 + \theta(a))e^{-\theta(a)} - (1 + \theta_0)e^{-\theta_0}](v - c_R) \quad (\text{A.35})$$

Attracting anxious sellers to the new submarket and violating part 2 of Definition 3.2 requires

$$\bar{V}_A \equiv [1 - (1 + \theta_0)e^{-\theta_0}](v - c_A) + \theta_0 e^{-\theta_0}(c_R - c_A) < -z + [1 - (1 + \theta(a))e^{-\theta(a)}](v - c_A)$$

or

$$\begin{aligned} z &< [(1 + \theta(a))e^{-\theta(a)} - (1 + \theta_0)e^{-\theta_0}](v - c_A) - \theta_0 e^{-\theta_0}(c_R - c_A) \\ &= [(1 + \theta(a))e^{-\theta(a)} - (1 + \theta_0)e^{-\theta_0}](v - c_R) + [e^{-\theta_0} - (1 + \theta(a))e^{-\theta(a)}](c_R - c_A) \end{aligned} \quad (\text{A.36})$$

Finding a z that satisfies (A.35) and (A.36) requires $\exp(\theta(a) - \theta_0) > 1 + \theta(a)$. Applying the free entry conditions to substitute for θ_0 and $\theta(a)$ yields

$$\kappa_0 > \sigma(a)(v - c_A) \exp\left(\frac{v - c_R - \sigma(a)(v - c_A)}{v - c_R}\right) \quad (\text{A.37})$$

However, given that $\sigma(a) \in ((v - c_R)/(v - c_A), 1]$, there does not exist a $\kappa_0 < \underline{\kappa}$ satisfying (A.37). There are no deviations that violate part 2 of Definition 3.2 and thus the full pooling submarket constitutes an equilibrium. $\square$

Proof of Proposition 3.3. Consider a listing agreement designed for type A sellers with the list price $p_A = c_A/(1 - z_A)$. The RWA clause has no effect on buyers' bidding strategies in a type A submarket, and does not impact anxious sellers' expected payoff. Type R sellers, on the other hand, now face a penalty equal to $\min\{z_A c_A/(1 - z_A), c_R - c_A\}$ in a bilateral match in the type A submarket.¹⁹

Recall the binding type R incentive compatibility constraint in the absence of a RWA clause:

$$\begin{aligned} V_R(0, 0, \theta_R) &= [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) \\ &= [1 - (1 + \theta_A)e^{-\theta_A}] [(1 - z_A)v - c_R] = V_R(z_A, 1, \theta_A) \end{aligned} \quad (\text{A.38})$$

With the RWA clause, this same condition becomes

$$\begin{aligned} &[1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) \\ &> [1 - (1 + \theta_A)e^{-\theta_A}] [(1 - z_A)v - c_R] - \theta_A e^{-\theta_A} \min\left\{\frac{z_A c_A}{1 - z_A}, c_R - c_A\right\} \end{aligned} \quad (\text{A.39})$$

The extra term on the right hand side indicates that the inequality is no longer binding. Commission rates less than z_A are incentive compatible for relaxed sellers. $\square$

¹⁹The minimization operator reflects the mimicking seller's choice between rejecting the offer but paying the REA's commission, $z_A c_A/(1 - z_A)$, and going ahead with the transaction at a price below their reservation value.

B Idiosyncratic Match Quality (For Online Publication)

The purpose of this appendix is to show that ex post heterogeneity on the demand side of the market does not generate endogenous market separation in the version of the model without real estate agents. Rather than buyers with common values as in Section 2, suppose that upon visiting a seller, the value that a buyer assigns to owning the home is a match specific random variable to reflect the idiosyncratic quality of the match. The random variable might capture the fact that houses are “inspection” goods, or “search” goods as in Nelson (1970). The subtle differences between units that are only observable by visiting and inspecting a house result in variation in buyers’ ex post valuations. Assume the match-specific valuation, v , is known only to the buyer, and is an independent draw from a standard uniform distribution.

If a buyer meets a seller and a transaction takes place at price p , the payoff to the buyer is $v - p$, and the payoff to the seller is $p - c$, where $v \in [0, 1]$ refers to the quality of the match between the buyer and the house, and $c \in \{c_A, c_R\}$ refers to the reservation value of the seller. For convenience, normalize $c_A = 0$ and $c_R = c \in (0, 1)$.

B.1 Buyers’ Bidding Strategies

Consider a housing market characterized by the buyer-seller ratio θ , and the fraction of motivated sellers σ . Let $b_k(v)$ denote buyers’ symmetric, increasing, and differentiable bidding strategy in a match with k other buyers. In equilibrium, it is optimal for a bidder with value v to submit $b = b_k(v)$ if all other buyers visiting the same seller submit bids according to $b_k(\cdot)$. Bidding b' in a match with k other buyers yields expected payoff

$$(b_k^{-1}(b'))^k \times \begin{cases} \sigma(v - b') & \text{if } b' < c \\ v - b' & \text{if } b' \geq c \end{cases}$$

Maximizing with respect to b' yields a first-order condition. To impose symmetric equilibrium bidding behaviour, the first-order condition is evaluated at $b' = b_k(v)$, which yields

$$\frac{d}{dv}b_k(v) + \frac{kb_k(v)}{v} = k$$

Since $b_k(0) = 0$,

$$b_k(v) = \frac{k}{k+1}v, \quad v \in [0, \hat{v})$$

where $\hat{v}$ satisfies

$$\sigma \left(\hat{v} - \frac{k}{k+1}\hat{v} \right) = \hat{v} - c \quad \Rightarrow \quad \hat{v} = \left(\frac{k+1}{k+1-\sigma} \right) c$$

which follows from the other boundary condition, $b_k(\hat{v}) = c$. The bidding function is therefore

$$b_k(v) = \begin{cases} \frac{k}{k+1}v & \text{if } v \in \left[0, \frac{(k+1)c}{k+1-\sigma} \right) \\ \frac{k}{k+1}v + \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{(k+1-\sigma)v} \right]^{k+1} v & \text{if } v \in \left[\frac{(k+1)c}{k+1-\sigma}, 1 \right] \end{cases} \quad (\text{B.1})$$

B.2 Expected Payoffs and Free Entry

The expected payoff to a buyer with value v in a match with k other buyers is

$$U(v, \sigma, k) = \begin{cases} \frac{\sigma v^{k+1}}{k+1} & \text{if } v \in \left[0, \frac{(k+1)c}{k+1-\sigma} \right) \\ \frac{v^{k+1}}{k+1} - \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+1} & \text{if } v \in \left[\frac{(k+1)c}{k+1-\sigma}, 1 \right] \end{cases} \quad (\text{B.2})$$

For a buyer who has visited a seller and observed the number of other bidders, but has yet to inspect the house and draw a value, the expected payoff is

$$\begin{aligned}
U(\sigma, k) &= \int_0^{\frac{(k+1)c}{k+1-\sigma}} \frac{\sigma v^{k+1}}{k+1} dv + \int_{\frac{(k+1)c}{k+1-\sigma}}^1 \left(\frac{v^{k+1}}{k+1} - \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+1} \right) dv \\
&= \frac{1}{(k+2)(k+1)} + \frac{1-\sigma}{k+2} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+2} - \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+1}
\end{aligned} \tag{B.3}$$

Prior to visiting the seller, the buyer's expected payoff is

$$U(\sigma, \theta) = e^{-\theta} \sum_{k=0}^{\infty} \frac{\theta^k}{k!} U(\sigma, k) \tag{B.4}$$

For example, if $\sigma = 1$ and $\theta = \theta_A$ (a market with only anxious sellers) the buyer's expected payoff from visiting a seller is

$$U(1, \theta_A) = e^{-\theta_A} \sum_{k=0}^{\infty} \frac{\theta_A^k}{(k+2)!} = \frac{1 - (1 + \theta_A)e^{-\theta_A}}{\theta_A^2} \tag{B.5}$$

and if $\sigma = 0$ and $\theta = \theta_R$ (a market with only relaxed sellers) the expected payoff is

$$\begin{aligned}
U(0, \theta_R) &= e^{-\theta_R} \sum_{k=0}^{\infty} \frac{\theta_R^k}{(k+2)!} [1 + (k+1)c^{k+2} - (k+2)c^{k+1}] \\
&= \frac{1 - [1 + \theta_R(1-c)] e^{-\theta_R(1-c)}}{\theta_R^2}
\end{aligned} \tag{B.6}$$

A relaxed seller's expected payoff when matched with k buyers is

$$\begin{aligned}
V_R(\sigma, k) &= \int_{\frac{kc}{k-\sigma}}^1 (k) v^{k-1} (b_{k-1}(v) - c) dv \\
&= \frac{k-1}{k+1} + \frac{c}{k+1} \left(\frac{kc}{k-\sigma} \right)^k + (1-\sigma) \left(\frac{kc}{k-\sigma} \right)^k \left[1 - \frac{k}{k+1} \left(\frac{kc}{k-\sigma} \right) \right] - c
\end{aligned} \tag{B.7}$$

which is the expectation of the highest bid less the reservation value, conditional on the highest bid exceeding c (i.e., conditional on the highest valuation exceeding $kc/(k-\sigma)$).

Conditioning on a high enough bid reflects the absence of commitment; every offer below c is costlessly rejected by a relaxed seller.

Prior to buyer arrival, the expected payoff takes into account the matching probabilities:

$$V_R(\sigma, \theta) = e^{-\theta} \sum_{k=2}^{\infty} \frac{\theta^k}{k!} V_R(\sigma, k) \quad (\text{B.8})$$

In a market with only relaxed sellers, the seller's expected payoff is

$$\begin{aligned} V_R(0, \theta_R) &= e^{-\theta_R} \sum_{k=2}^{\infty} \frac{\theta_R^k}{k!} \left(\frac{k-1}{k+1} [1 - c^{k+1}] + c^k - c \right) \\ &= (1-c) \left[1 + e^{-\theta_R(1-c)} \right] - \frac{2}{\theta_R} \left[1 - e^{-\theta_R(1-c)} \right] \end{aligned} \quad (\text{B.9})$$

whereas deviating to a market with only anxious sellers yields an expected payoff equal to

$$V_R(1, \theta_A) = e^{-\theta_A} \sum_{k=2}^{\infty} \frac{\theta_A^k}{k!} \left(\frac{k-1}{k+1} [1 - c^{k+1}] + \frac{c^{k+1}}{k+1} \left[k - 1 + \left(\frac{k}{k-1} \right)^k \right] - c \right) \quad (\text{B.10})$$

B.3 The Non-Existence of a Separating Equilibrium

I demonstrate numerically the non-existence of a separating equilibrium in the version of the model without REAs. For a grid of values for $c \in (0, 1)$, I first use the free entry conditions, $U(1, \theta_A) = \kappa_0$ and $U(0, \theta_R) = \kappa_0$ to check that both submarkets are active (i.e., to verify that buyers find it worthwhile to enter both markets) and to calculate the buyer-seller ratios, θ_A and θ_R . If κ_0 is low enough for both markets to attract buyers, I compute $V_R(0, \theta_R)$ and $V_R(1, \theta_A)$ using (B.9) and (B.10). I then show that $V_R(1, \theta_A) > V_R(0, \theta_R)$, which violates incentive compatibility. I repeat this exercise for different values of κ_0 . Figures 3 and 4 illustrate the incentive for relaxed sellers to deviate from a market with only relaxed sellers to a market with only anxious sellers.

The intuition for incentive incompatibility is similar to the model without ex post buyer heterogeneity (see Proposition 2.2). The free entry conditions imply $\theta_A > \theta_R$. The market

for anxious sellers therefore has higher matching probabilities and more intense competition among bidders. It is the appeal of higher offers without requiring commitment to sell when only low offers are received that precludes incentive compatibility.

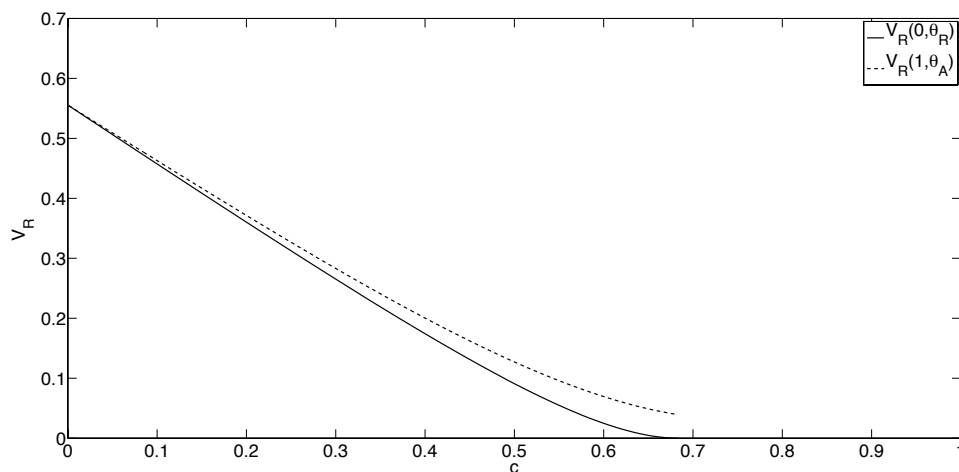


Figure 3: A relaxed seller's expected payoff with market separation ($\kappa_0 = .05$).

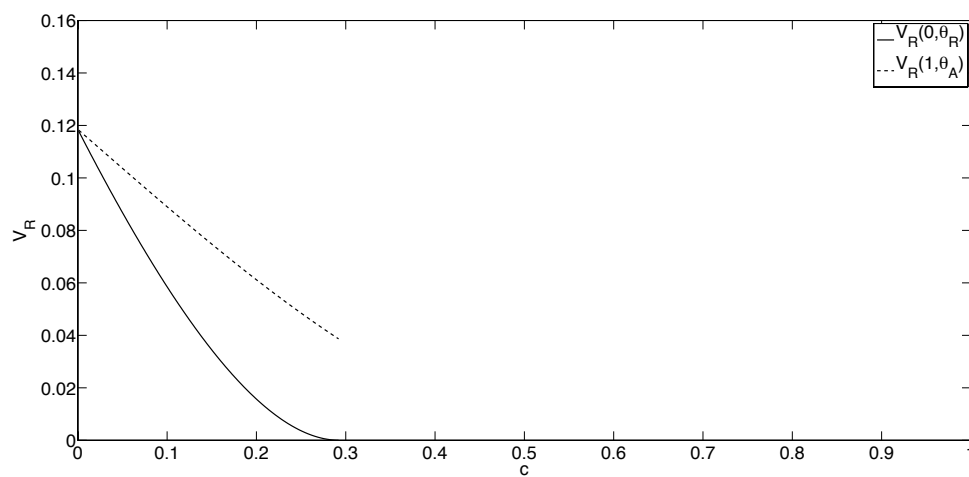


Figure 4: A relaxed seller's expected payoff with market separation ($\kappa_0 = .25$).