

Commitment and Costly Signalling in Decentralized Markets

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Abstract

I propose a search model of a decentralized market with asymmetric information in which sellers are unable to commit to asking prices announced *ex ante*. Relaxing the commitment assumption prevents sellers from using price posting as a signalling device to direct buyers' search. Private information about the gains from trade and inefficient entry on the demand side then contribute to market illiquidity. Endogenous sorting among costly marketing platforms can facilitate the search process by segmenting the market to alleviate information frictions. Seemingly irrelevant but incentive compatible listing fees are implementable provided that the market is not already sufficiently active.

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1 Introduction

In this paper, I develop a directed search model of a decentralized market with private information in which sellers lack the ability to commit to asking prices. Under certain assumptions about the matching process and the method of price determination, the absence of commitment precludes asking prices as an effective means of disclosing sellers' unobservable reservation values to appropriately direct buyers' search. Instead, the net social surplus from market transactions is adversely affected by information frictions and inefficient entry of buyers. In some circumstances, however, sellers of different types can reveal their willingness to sell and attract the appropriate number of buyers by sorting endogenously among costly marketing platforms. Otherwise wasteful listing fees can therefore play a role in overcoming information frictions and improving market efficiency.

I consider a decentralized market in which sellers advertise a single indivisible and homogeneous item for sale to attract buyers. Each buyer can incur a cost to visit a seller, but the matching process is subject to search or coordination frictions resulting in instances where multiple buyers visit the same seller. Prices are determined by the equilibrium bidding strategies of buyers when the number of competing bidders in a match is observable. A seller's reservation value is the crucial source of private information. I show that non-binding list prices as cheap talk are uninformative. Sellers with high reservation values mimic sellers with low reservations values in order to drive up the final sale price by increasing the probability of a bidding war. This result relies on the inability to commit to *ex ante* prices and, to some extent, the particulars of the matching process and pricing protocol. The intuition is as follows. The appeal of a higher expected selling price in multiple offer situations is offset by the possibility of a single low offer in a bilateral match. Without commitment to an asking price, however, a seller is free to reject any unacceptable offer and is therefore primarily concerned about increasing the expected number of buyer visits, regardless of her reservation value.

I propose a costly signalling mechanism for market separation: if sellers can sort endogenously among costly marketing platforms, I find that in some circumstances multiple platforms are incentive compatible and implement a separating directed search equilibrium. This alleviates the information problem and increases activity in the market. Even though marketing costs in the form of listing fees provide no technological advantage in the matching process, incentive compatible market separation is achievable as long as search costs are high enough that the market would otherwise be sufficiently inactive. In high-demand settings, on the other hand, motivated/anxious sellers cannot justify allocating resources to a costly signalling technology to further increase the expected number of buyer visits.

Endogenous sorting among costly platforms is shaped by anxious sellers' pre- and post-match incentives to separate. *Ex ante*, anxious sellers might be willing to pay a listing fee if it attracts more buyers (in expectation) by signalling a higher surplus from trade. Since sellers respond differently to changes in the expected number of buyer visits, which in turn is related to the endogenous composition of sellers on a particular platform, it can be self-fulfilling for anxious sellers to advertise on a platform that is costly enough to discourage mimicking by less motivated sellers. *Ex post*, however, sellers with low reservation values prefer situations with private information because, in a bilateral match, the buyer submits a better offer if his beliefs assign a sufficiently high probability to the possibility that the seller's reservation value is high. In general, the first effect dominates and market separation emerges as buyers become scarce: *i.e.*, as search costs increase and fewer buyers choose to participate in the market.

This paper is part of the search literature, and in particular contributes to the study of markets with search frictions and private information. In contrast to existing theories of competitive or directed search with private information that rely on strong commitment assumptions,² relaxing the assumption of full commitment to the announced terms of trade is

²Recent papers in this literature include Guerrieri et al. (2010) and Delacroix and Shi (2013), to name a few. Guerrieri et al. (2010) present a search environment with adverse selection and show that screening

an important element in this paper. A related paper by Kim (2012) shows that non-binding messages can support a partially separating equilibrium in a decentralized asset market when there is private information about the quality of the asset. Non-binding list prices are ineffective here because it is the seller’s reservation value that is private information, which is independent of the buyer’s valuation. Menzio (2007) relaxes the commitment assumption in a model of the labour market and shows that cheap talk can sometimes credibly convey information when wages are determined through bargaining. In a partially directed search equilibrium, a deviation from truth-telling that improves the matching probability will result in a lower negotiated share of the surplus. In an environment with auctions, in contrast, the transaction price increases with the number of buyers in a match.³ Without commitment to an asking price, sustaining a fully revealing and constrained efficient equilibrium requires a trading protocol that does not generate too much price dispersion (Kim and Kircher, 2014). Otherwise, the absence of commitment hinders truth-telling and unravels market separation, as I show in the version of the model without marketing platforms that charge listing fees.

Costly signalling has been studied extensively in the literature without search frictions. In Spence’s (1973) canonical model of labour market signalling, a productive worker undertakes a costly activity such as acquiring education to distinguish himself from a less-productive worker. A crucial assumption is that the worker’s cost of undertaking the signalling activity is negatively correlated with his unobservable productivity so that signalling can be an effective way to distinguish worker types. This type of *single-crossing property* is often assumed in signalling theory. Fang (2001) constructs a signalling model that generates an *endogenous* single-crossing property by allowing uninformed firms to give preferential treatment to, and form different beliefs about the group of workers that undertake the signalling activity. The

results in a fully-revealing competitive search equilibrium when the uninformed party can commit to a take-it-or-leave-it trading mechanism. Delacroix and Shi (2013) study a model with adverse selection where sellers can post non-negotiable prices as a means of directing search, and also as a signal of the quality of their asset.

³Julien et al. (2006) highlight the implications of this type of setup for residual price dispersion in a theory of the labour market with full information.

model presented in this paper implements a similar mechanism in a decentralized market with search frictions; costly signalling is embedded into the choice of submarket in a directed search equilibrium. Equilibrium separation of seller types with costly marketing platforms is then related to papers in the signalling literature that identify an indirect signalling role of conspicuous advertising expenditures when producer quality is unobservable (Nelson, 1974; Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986; Bagwell and Ramey, 1994).

Consider an application of the model to housing markets. Search frictions, seller heterogeneity, multiple offers by competing bidders, and non-binding list prices are common and well-documented phenomena in the context of housing markets.⁴ The absence of commitment to posted prices is particularly relevant; in Canada and the U.S., there is no legal obligation associated with the list price that compels a seller to accept an offer. The theoretical results pertaining to costly signalling provide insight about the surprisingly high cost of real estate agents and the coexistence of multiple marketing platforms. More specifically, the theoretical predictions are consistent with the recent empirical evidence of endogenous sorting between full-commission full-service realtors, and low-cost limited-service agents (Bernheim and Meer, 2008; Levitt and Syverson, 2008a; Hendel et al., 2009): sellers represented by full-commission agents tend to exhibit characteristics consistent with high motivation to sell, and consequently experience shorter selling times and a higher probability of sale.

The next section presents the model of a decentralized market with heterogeneity in seller motivation but without costly signalling. A comparison of the market equilibrium with the constrained efficient allocation leads to a discussion of how information frictions give rise to market *illiquidity* (*i.e.*, a less active market). The costly signalling technology is introduced in Section 3. The theoretical results are discussed in the context of housing markets in

⁴There is a large literature that applies search theory to model the housing market (Wheaton, 1990; Krainer, 2001; Albrecht et al., 2007; Díaz and Jerez, 2013; Head et al., 2014). Albrecht et al. (2015) and Han and Strange (2014) focus on multiple offers by competing bidders and emphasize limited commitment to the asking price of a house. Finally, Glower et al. (1998) conduct a survey of home sellers and find substantial heterogeneity in terms of motivation to sell.

Section 4. Section 5 concludes.

2 The Model

Language related to housing markets is used to describe the model because of the application in Section 4. The model could similarly be applied to decentralized markets for goods, assets or labour.

Agents, endowments and preferences. There is a fixed measure S of sellers, and a measure B of buyers determined by free entry. Each seller owns one indivisible house, and each buyer has unit demand for a house. Heterogeneity on the supply side reflects differences in willingness to sell: a fraction $\hat{\sigma}$ of sellers have a low opportunity cost of selling or reservation value, c_A , while the remaining $1 - \hat{\sigma}$ of sellers have a high reservation value, $c_R > c_A$. Type A and R sellers are labelled *anxious* and *relaxed* since, in a dynamic setting, differences in reservation values could reflect varying degrees of patience; more patient or relaxed sellers are more inclined to turn down low offers and wait for more favourable terms of trade in the future. Sellers' reservation values are private information, but the proportion of anxious sellers is commonly known. Buyers are homogeneous, and assign value $v > c_R$ to home ownership.⁵

Buyers and sellers are risk neutral. If a buyer purchases a house at price p , the payoff to the buyer is $v - p$, and the payoff to the seller is $p - c$, where $c \in \{c_A, c_R\}$ refers to the reservation value of the seller. A buyer's payoff if he fails to purchase a house is normalized to 0.

Search and matching. Each potential buyer can pay a cost $\kappa < v - c_R$ to enter the market and visit exactly one seller, but the matching process is subject to frictions.

⁵While *ex post* buyer heterogeneity would add another layer of generality to the model without affecting the main results, omitting match specific values reduces the risk of obscuring the central arguments of the paper. I show in Appendix B that introducing *ex post* buyer heterogeneity would not automatically generate endogenous market separation. It would, however, add to the analytical complexity when costly marketing platforms are introduced in Section 3.

Specifically, the probability that a seller is matched with exactly k buyers follows a Poisson distribution,

$$e^{-\theta} \cdot \frac{\theta^k}{k!}, \quad k = 0, 1, \dots$$

where θ is the endogenously determined ratio of buyers to sellers, termed *market tightness*.⁶ Meetings between buyers and sellers arise in a setting with directed search wherein buyers can condition their entry/search decisions on sellers' observable characteristics. While this distinction is irrelevant when sellers are indistinguishable from one another, it is important when reservation values are observable and when environments with potentially effective signalling devices are considered. In such cases, multiple submarkets can arise endogenously so that tightness in each submarket is determined by the equilibrium strategies of buyers and sellers.

Price determination. In a match between a seller and k buyers, each buyer makes a take-it-or-leave-it offer after observing the number of competing bidders. The seller is free to accept the highest bid or reject all offers. In the event that there are identical bids for an amount that is both highest and acceptable, the seller randomly selects among these offers, so that each bidder has an equal probability of purchasing the home.

The assumption that a buyer observes the presence of other buyers at a particular seller is important for the results that follow.⁷ Moreover, it differs from the trading protocol emphasized in Kim and Kircher (2014), where the number of competing bidders in a first-price auction is assumed to be *unobservable*. The results derived in this section, however, are

⁶This matching process often emerges in the literature in models of economies with coordination frictions and a finite number of sellers and buyers with symmetric search strategies. The Poisson matching probabilities are calculated for a large market with $B, S \rightarrow \infty$ and $B/S = \theta$ (see Butters, 1977; Burdett et al., 2001).

⁷This assumption is consistent with the reported experiences of recent home buyers according to a survey conducted by Genesove and Han (2011). In any case, buyers merely need to know when there is at least one competing bidder, which is information that the seller would choose to disclose given that the presence of other buyers escalates the price of the house. If there does not exist a credible method for doing so, permitting buyers to submit bids with escalator clauses would render moot the issues of observable competitors and credible disclosure.

relatively general in that they apply to settings where market participation does not entail commitment to transact and the price is sufficiently sensitive to the number of bidders.

Strategies and payoffs. A seller's optimal acceptance decision is to accept the highest offer if and only if it is greater than or equal to her reservation value, $c \in \{c_A, c_R\}$. Buyers' bidding strategies depend on the number of competing bidders and the composition of sellers in the submarket, σ . In a bilateral match, the buyer is free to make an offer without worrying about competing bidders. In such cases, the solitary buyer offers either c_A or c_R , whichever yields the highest expected payoff. If $\sigma(v - c_A) > v - c_R$, the buyer offers c_A knowing that if the seller is relaxed, the offer is rejected and there is no transaction. Otherwise, if $\sigma(v - c_A) < v - c_R$, the buyer sensibly offers c_R , and trade will occur regardless of the seller's type. If $\sigma(v - c_A) = v - c_R$, the buyer is indifferent between bidding c_A and c_R and may implement a mixed strategy. When more than one buyer visits the same seller (a multilateral match), they compete à la Bertrand and bid their valuation, v .

Lemma 1. *A Seller accepts the highest offer if and only if it is greater than or equal to her reservation value, $c \in \{c_A, c_R\}$. Buyers bid their valuation, v , in a multilateral match. In a bilateral match, he bids c_A if $\sigma(v - c_A) > v - c_R$, c_R if $\sigma(v - c_A) < v - c_R$, or c_R and c_A with probabilities $\pi \in [0, 1]$ and $1 - \pi$ if $\sigma(v - c_A) = v - c_R$.*

When the buyer-seller ratio is θ , the fraction of anxious sellers is σ , and the bidding and acceptance strategies follow Lemma 1, the expected payoff to a buyer is

$$U(\sigma, \theta) = e^{-\theta} \max \{ \sigma(v - c_A), v - c_R \} = \begin{cases} e^{-\theta} \sigma(v - c_A) & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\ e^{-\theta} (v - c_R) & \text{if } \sigma \leq \frac{v - c_R}{v - c_A} \end{cases} \quad (1)$$

This is the payoff in a bilateral situation, which occurs with probability $e^{-\theta}$. The expected payoff in a multilateral match with $k \geq 1$ other buyers is zero since the equilibrium bid is v . Two cases arise because the share of anxious sellers determines whether c_A or c_R is offered in a bilateral match, as described in Lemma 1.

The expected payoff to a relaxed seller is

$$V_R(\theta) = e^{-\theta} \sum_{k=2}^{\infty} \frac{\theta^k}{k!} (v - c_R) = [1 - (1 + \theta)e^{-\theta}] (v - c_R) \quad (2)$$

The final expression recognizes the McLaurin series of the exponential function. The simplicity of this expression arises because the payoff to a type R seller in a bilateral match is zero regardless of whether or not a transaction takes place. An anxious seller, on the other hand, has the following expected payoff:

$$V_A(\sigma, \theta) = [1 - (1 + \theta)e^{-\theta}] (v - c_A) + \begin{cases} 0 & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\ \pi \theta e^{-\theta} (c_R - c_A) & \text{if } \sigma = \frac{v - c_R}{v - c_A} \\ \theta e^{-\theta} (c_R - c_A) & \text{if } \sigma < \frac{v - c_R}{v - c_A} \end{cases} \quad (3)$$

The last term reflects the positive surplus for a type A seller in a bilateral match whenever the buyer offers $c_R > c_A$.

Lemma 2. *Expected payoffs exhibit the following properties:*

- (i) U is strictly decreasing in θ . U is strictly increasing in σ if $\sigma > (v - c_R)/(v - c_A)$ and constant if $\sigma \leq (v - c_R)/(v - c_A)$.
- (ii) V_R is strictly increasing in θ and independent of σ .
- (iii) V_A is strictly increasing in θ . V_A decreases by $\theta e^{-\theta} (c_R - c_A)$ at $\sigma = (v - c_R)/(v - c_A)$ but remains constant otherwise.

Equilibrium. The free entry of buyers, $U(\sigma, \theta) = \kappa$, determines the equilibrium buyer-seller ratio(s). The visibility of seller characteristics determines whether buyers can direct their search and segment the market.

Definition 2.1. *A market equilibrium with private information is a market tightness, θ , satisfying the free entry condition, $U(\hat{\sigma}, \theta) = \kappa$. A market equilibrium under full information is a pair $\{\theta_A, \theta_R\}$ satisfying $U(1, \theta_A) = U(0, \theta_R) = \kappa$.*

If sellers' reservation values are observable, buyers condition their search strategy and bilateral offers on the seller's willingness to sell. The expected payoffs to buyers in a market with observable c_A and c_R determine $\{\theta_A, \theta_R\}$ according to the free entry conditions:

$$U(1, \theta_A) = e^{-\theta_A}(v - c_A) = \kappa \quad (4)$$

$$U(0, \theta_R) = e^{-\theta_R}(v - c_R) = \kappa \quad (5)$$

In contrast, the equilibrium with unobservable reservation values resembles a random search equilibrium with both types of sellers attracting buyers in a single market. Market tightness in equilibrium, θ , is determined by a single free entry condition:

$$U(\hat{\sigma}, \theta) = e^{-\theta} \max \{ \hat{\sigma}(v - c_A), v - c_R \} = \kappa \quad (6)$$

2.1 Information and Efficiency

Consider a constrained social planner that seeks to maximize *net social surplus*, defined as the total social surplus from all market transactions less search costs. Choice among allocations is *constrained* in the sense that the planner is subject to the same search and information frictions faced by market participants. It is of interest to then compare this constrained efficient allocation to the decentralized equilibrium.

With full information, the pricing mechanism ensures that the gains from transferring ownership of a house are realized in every match. Moreover, buyers face undistorted incentives in searching for a house when sellers' reservation values are discernible.

Proposition 2.1. *The market equilibrium under full information is constrained efficient.*

The proof of Proposition 2.1 shows that the optimality conditions associated with the planner's problem coincide exactly with the free entry conditions for buyers in markets with full information. All proofs are relegated to Appendix A.

In contrast to the full information equilibrium, market *illiquidity* (*i.e.*, a reduction in market activity) arises with private information due to its interaction with both the pricing mechanism and buyer entry. Figure 1 illustrates market liquidity (as measured by the average probability of a transaction) in terms of the composition of sellers. When $\hat{\sigma}$ is high, $\hat{\sigma} > (v - c_R)/(v - c_A)$, the information frictions impede mutually beneficial transactions when buyers make take-it-or-leave-it offers in bilateral matches that get rejected whenever the seller is less motivated to sell. Even when $\hat{\sigma}$ is low, $\hat{\sigma} \leq (v - c_R)/(v - c_A)$, private information about sellers' reservation values reduces market liquidity by affecting the incentive to search. When buyers offer $c_R > c_A$ in a bilateral match and their share of the surplus in a transaction with an anxious seller is reduced, too few buyers find it worthwhile to participate in the market by visiting a house for sale. This is an implication of the free entry condition.

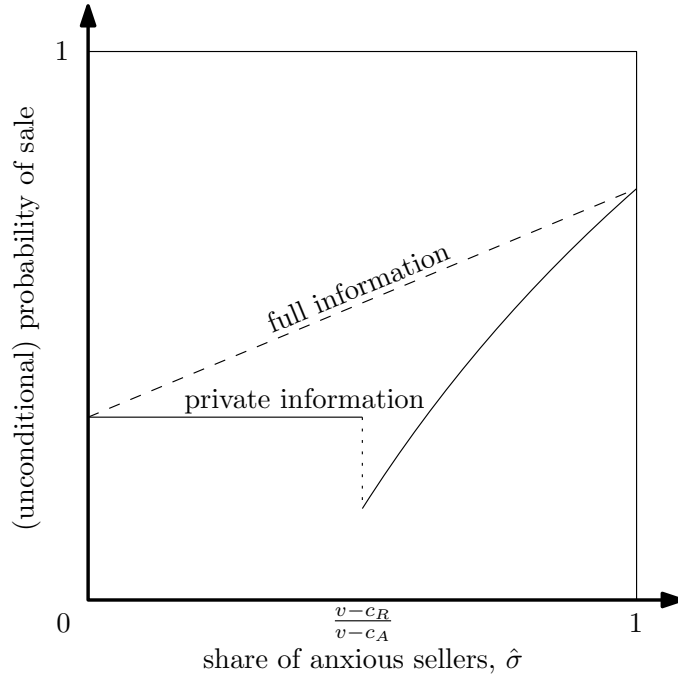


Figure 1: Market liquidity with private information relative to the (constrained efficient) case with full information.

2.2 Non-Binding List Prices

It has been established that it is desirable from liquidity and efficiency perspectives for sellers with different reservation values to be distinguishable. If sellers could differentiate themselves by announcing cheap-talk messages in the form of non-binding list prices, for example, then buyers would direct their search. More buyers would visit the anxious sellers, knowing that a lower offer is accepted in a bilateral match. The list price as a non-contractual or cheap-talk message, however, is not a credible signalling device: relaxed sellers will list their homes to mimic anxious sellers in order to attract more buyers. This increases the probability that a bidding war will drive the selling price up to v . This result is stated formally in Proposition 2.2.

Proposition 2.2. *If $\{p_1, p_2\}$ denote any two non-binding list prices announced by sellers and $\{\theta_1, \theta_2\}$ the respective buyer-seller ratios in equilibrium, then $\theta_1 = \theta_2$.*

A correlation between the non-binding list price and the seller's reservation value is unsustainable, and search remains undirected in equilibrium owing to uninformative list prices. Without the ability to commit to a list price, market separation violates incentive compatibility. Note that the incentive for a relaxed seller to mimic an anxious seller is clear in the current setup given the assumption of take-it-or-leave-it offers.⁸ More generally, the result in Proposition 2.2 maintains in settings with two important features: (i) the inability to commit *ex ante* to the terms of trade, and (ii) a method of price determination such that the expected selling price in multiple offer situations is sufficiently high. This second feature makes it appealing for a relaxed seller to underrepresent her reservation value. Doing so may also invite low and even unacceptable offers from bidders; however, the freedom to abstain from selling when only low offers are received mitigates this effect which impedes incentive compatible market separation. The specific method of price determination studied

⁸In the event of a bilateral match, a relaxed seller's payoff is zero regardless of whether the buyer offers c_R (leaving the seller with none of the surplus) or c_A (in which case the relaxed seller simply rejects the offer).

in this paper is one that satisfies these criteria, preserves tractability, and replicates features of trading protocols that commonly emerge in housing and other decentralized markets.

A brief discussion of the related literature is in order. Albrecht et al. (2015) show that even partial commitment to an asking price (specifically, only offers below the posted price can be rejected) is sufficient for achieving constrained efficiency in a setting where matching is multilateral and auctions determine the terms of trade. Two papers that fully relax price commitment are Menzio (2007) and Kim and Kircher (2014). Menzio (2007) shows that non-contractual messages in job listings can sometimes credibly convey information when wages are determined through bilateral bargaining. The particulars of the bilateral bargaining protocol rule out the possibility that a deviation from truth-telling can improve both the expected share of the surplus and the probability of a match. Kim and Kircher (2014) show in a similar environment to this paper that efficiency can be sustained without commitment to reservation prices for a class of trading protocols that includes first-price auctions where bidders submit offers *before* observing the number of competitors. In contrast, the method of price determination here is such that falsely advertising a larger surplus improves the probability of multiple bidders without markedly reducing the aggressiveness of their offer strategy. More generally, market separation becomes unsustainable without commitment to list prices as the expected payoff to the seller relies less on the share of the surplus captured in a bilateral match and more on price escalation with multiple buyers.

3 Costly Signalling

This section investigates whether endogenous sorting among costly marketing platforms can fulfil an informational role and achieve market separation. The availability of different marketing platforms that charge listing fees to sellers can be interpreted as a costly signalling technology embedded in a directed search framework. Despite the irrelevance and wastefulness of listing fees in the absence of information frictions, access to a money burning

technology may induce market separation, allow buyers to direct their search, and help overcome the problems related to private information and the absence of commitment.

More specifically, suppose there exists a continuum of marketing platforms that charge upfront non-refundable fees $x \in \mathbb{R}_+$. Paying $x > 0$ has no direct benefit to potential buyers or to the seller, but with x observable it becomes feasible for sellers to burn money as a means of conveying their type. Sellers sort themselves into submarkets by paying the required fee to list their house for sale on that platform, and buyers enter submarkets by choosing among marketing platforms that are identifiable by their listing fee. Each buyer then visits a single seller among those participating in the same submarket/platform. When buyers match with sellers, they implement competitive bidding strategies to purchase the house and sellers make offer acceptance decisions according to Lemma 1.

Definition 3.1. *A market equilibrium with private information and costly marketing platforms is a set of active platforms $\mathbb{X} \subset \mathbb{R}_+$, a distribution of buyers λ on $\mathbb{R}_+$ with support $\mathbb{X}$, a function $\theta : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \cup \{+\infty\}$, a function $\sigma : \mathbb{R}_+ \rightarrow [0, 1]$, and a pair of seller values $\{\bar{V}_A, \bar{V}_R\}$ satisfying*

1. *anxious sellers' optimal sorting:* $\forall x \in \mathbb{R}_+$,

$$\bar{V}_A \geq V_A(\sigma(x), \theta(x)) - x,$$

with equality if $\theta(x) < \infty$ and $\sigma(x) > 0$, where

$$\bar{V}_A = \max_{x \in \mathbb{X}} \{V_A(\sigma(x), \theta(x)) - x\};$$

2. *relaxed sellers' optimal sorting:* $\forall x \in \mathbb{R}_+$,

$$\bar{V}_R \geq V_R(\theta(x)) - x,$$

with equality if $\theta(x) < \infty$ and $\sigma(x) < 1$, where

$$\bar{V}_R = \max_{x \in \mathbb{X}} \{V_R(\theta(x)) - x\};$$

3. *buyers' optimal entry/search*: $\forall x \in \mathbb{R}_+$,

$$U(\sigma(x), \theta(x)) \leq \kappa,$$

with equality if $x \in \mathbb{X}$; and

4. *market clearing*:

$$\int_{\mathbb{X}} \frac{\sigma(x)}{\theta(x)} d\lambda(x) = \hat{\sigma} S \quad \text{and} \quad \int_{\mathbb{X}} \frac{1 - \sigma(x)}{\theta(x)} d\lambda(x) = (1 - \hat{\sigma}) S.$$

The definition of an equilibrium is such that for every $x \in \mathbb{R}_+$, there is a $\theta(x)$ and a $\sigma(x)$. Part 4 of the definition makes certain that all sellers list on some marketing platform. Part 3 states that θ is derived from the free entry condition of buyers for active platforms and hence guarantees that tightness is consistent with optimal entry/search on the part of buyers. Similarly, parts 1 and 2 require that, for active platforms, σ is derived from the composition of sellers that find it optimal to use that platform. For inactive platforms, parts 1 and 2 further establish that θ and σ are determined by the optimal sorting of sellers so that off-equilibrium beliefs are pinned down by the following requirement: if a small measure of buyers could be enticed to search platform $x \notin \mathbb{X}$ and sellers optimally sort among x and the active platforms, then those sellers willing to accept the lowest buyer-seller ratio on platform x determine both the composition of sellers $\sigma(x)$ and the buyer-seller ratio $\theta(x)$.⁹

This refinement concept, which restricts the equilibrium functions θ and σ , is based on a notion of forward induction. Upon observing sellers listing on platform x , buyers reason about what must have happened at the sorting stage, presuming that sellers behaved rationally. Submarket tightness $\theta(x)$ must therefore justify platform x as a rational choice for anxious sellers if $\sigma(x) = 1$, for relaxed sellers if $\sigma(x) = 0$, and for both types if $\sigma(x) \in (0, 1)$. Next, if the free entry condition does not hold with equality at x , then no positive measure of buyers would choose x in an equilibrium and the platform is not active: $x \notin \mathbb{X}$.

⁹If neither type of seller finds x acceptable for any finite buyer-seller ratio, then $\theta(x) = \infty$, which is interpreted as no positive measure of sellers choosing x .

ℳ. Restricting off-equilibrium beliefs in this way is closely related to refinements that are common in the competitive search literature, but is based on forward induction rather than subgame perfection.^{10,11} This approach to embedding costly signalling into a directed search framework helps to rule out unreasonable equilibria in which anxious sellers' attempts to signal their type are arbitrarily interpreted by buyers as unintentional mistakes by relaxed sellers.

3.1 Equilibrium Characterization

A pooling allocation on platform $x_P = 0$, as in the equilibrium with private information from Section 2, would require $\sigma_P = \hat{\sigma}$, and θ_P satisfying $U(\sigma_P = \hat{\sigma}, \theta_P) = \kappa$. A separating allocation with platforms $x_R = 0$ and $x_A > 0$ would have $\sigma_R = 0$, $\sigma_A = 1$, with $\{\theta_R, \theta_A\}$ satisfying $U(\sigma_R = 0, \theta_R) = U(\sigma_A = 1, \theta_A) = \kappa$. For platform x_A to attract anxious sellers but not relaxed sellers, x_A must satisfy the following incentive compatibility constraints:

$$V_A(\sigma_A = 1, \theta_A) - x_A \geq V_A(\sigma_R = 0, \theta_R) \quad (7)$$

$$V_R(\theta_A) - x_A \leq V_R(\theta_R) \quad (8)$$

Let $\mathbb{X}_A$ denote the set of all $x_A \in \mathbb{R}_+$ satisfying (7) and (8). $\mathbb{X}_A$ therefore defines the set of incentive compatible platforms when only relaxed sellers list on the zero-fee platform.

¹⁰See Guerrieri et al. (2010) for an example of such a restriction in a competitive search model where the uninformed side of the market sorts first into submarkets, followed by the informed side of the market.

¹¹One could apply a refinement based on subgame perfection and obtain the same results by explicitly modelling competitive market-makers that first separate the market into submarkets by charging fees for operating a publicly observable money burning technology. The appeal of the market-maker interpretation is that search/marketing agencies tend to manifest in decentralized markets (real estate agents in housing markets, for example, or job search and recruitment agencies in labour markets). An interesting conclusion is that even ineffective marketing agents may become relevant in a decentralized market with private information.

Proposition 3.1. $\mathbb{X}_A$ is non-empty if and only if

$$\kappa \geq (v - c_A) \exp \left(-\frac{c_R - c_A}{v - c_R} \right). \quad (9)$$

Proposition 3.1 specifies the necessary and sufficient condition for the existence of a pair of incentive compatible platforms. Define

$$x_A^* \equiv V_R(\theta_A) - V_R(\theta_R), \quad (10)$$

which, if $\mathbb{X}_A$ is non-empty, is the least-cost incentive compatible platform. The next proposition establishes that platforms 0 and x_A^* are the only ones that can be active in any equilibrium.

Proposition 3.2. Let $\{\mathbb{X}, \theta, \sigma, \lambda, \bar{V}_A, \bar{V}_R\}$ be an equilibrium.

- (i) Take any $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) = 0$. Then $x = 0$ and $\theta(x) = \theta_R$.
- (ii) Take any $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) \in (0, 1)$. Then $x = 0$, $\sigma(x) \leq (v - c_R)/(v - c_A)$ and $\theta(x) = \theta_R$.
- (iii) Take any $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) = 1$. Then $x = x_A^*$ and $\theta(x) = \theta_A$.

Parts (i) and (ii) of Proposition 3.2 imply that any active platform that lists relaxed sellers must not charge a listing fee. Otherwise, if $x > 0$, there exists a deviation that achieves a higher payoff; if $\sigma(x) = 0$, then relaxed sellers could attract more buyers at a lower-cost platform and achieve a higher expected payoff than $\bar{V}_R$, and if $\sigma(x) \in (0, 1)$, anxious sellers could attract more buyers at a higher-cost platform and achieve a higher expected payoff than $\bar{V}_A$. Part (iii) of Proposition 3.2 states that if some anxious sellers manage to separate from relaxed sellers, they do so in the least costly manner (subject to incentive compatibility) by listing on platform x_A^* .

Proposition 3.3 (Separating Equilibrium). *If $\mathbb{X}_A$ is non-empty, there exists an equilibrium $\{\mathbb{X}, \theta, \sigma, \lambda, \bar{V}_A, \bar{V}_R\}$ with $\mathbb{X} = \{x_R = 0, x_A = x_A^*\}$, $\sigma(x_R) = 0$ and $\sigma(x_A) = 1$. Moreover, the equilibrium $\bar{V}_R$ and $\bar{V}_A$ are unique.*

Proposition 3.3 establishes existence and uniqueness of a separating equilibrium with costly marketing platforms when a pair of incentive compatible platforms, $x_R = 0$ and $x_A \in \mathbb{X}_A$, is available. More specifically, if $\mathbb{X}_A$ is non-empty then it contains the least costly incentive compatible platform, x_A^* , where anxious sellers list their homes to attract buyers in a separating equilibrium.

Market separation can only be incentive compatible if the benefit from signalling (increased tightness, θ , and hence a higher probability of a multilateral match with payoff $v - c_A$) outweighs both the listing cost and the foregone informational rent (the extra payoff of $c_R - c_A$ in a bilateral match) captured in the submarket with relaxed sellers and no listing fee.¹² The benefit derived from an increase in market tightness dominates in markets that are less liquid (*i.e.*, whenever the buyer-seller ratios of active submarkets are sufficiently low). When demand is too high, the benefit of further increasing market tightness by separating from relaxed sellers is insufficient to offset both the listing fee and the foregone bilateral surplus. The parameter most directly (but inversely) related to market tightness is κ , the entry cost for buyers. The intuition for Proposition 3.1 is that when κ is high enough to satisfy (9), buyers are scarce and the benefit from signalling seller motivation is appealing enough to offset both the opportunity cost and the direct cost required to discourage relaxed sellers from mimicking.

The endogenous market separation result is somewhat reminiscent of the cultural equilibrium in Fang (2001). In Fang's paper, social culture is a seemingly irrelevant activity

¹²Specifically, the net benefit from signalling is

$$\begin{aligned} V_A(1, \theta_A) - V_A(0, \theta_R) - x_A^* &= [(1 + \theta_R)e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - \theta_R e^{-\theta_R} (c_R - c_A) - x_A^* \\ &= [e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}] (c_R - c_A). \end{aligned}$$

that can be used as an endogenous signalling device to partially overcome an informational free-riding problem in an economy with a labour market and the choice between old and new production technologies. Here, if the parameters are conducive to separation, anxious sellers elect to list on a costly but seemingly irrelevant platform to signal higher gains from trade and attract more buyers (in expectation). The incentive to separate when search frictions are sufficiently severe (as per Proposition 3.1) is driven by the endogenous relationships between matching probabilities, expected prices, and buyers' beliefs about the group of sellers on a particular platform. The refinement embedded in the definition of a directed search equilibrium (Definition 3.1) guarantees that precisely platform x_A^* is selected by anxious sellers so that relaxed sellers are just indifferent between the two platforms, thus ruling out Pareto inferior separating equilibria.

If inequality (9) does not hold, a separating equilibrium is not feasible because anxious sellers would find it optimal to deviate to the market dominated by relaxed sellers since a fraction $(c_R - c_A)/(v - c_A)$ of the surplus can be captured even in a bilateral match. Instead, there are pooling and semi-pooling equilibria when $\mathbb{X}_A = \emptyset$ for certain parameters. Propositions 3.4 and 3.5 fill in these details by characterizing the market equilibrium in terms of $\hat{\sigma}$ when κ does not satisfy (9). Figure 2 provides a graphical representation.

Proposition 3.4 (Pooling Equilibrium). *If $\mathbb{X}_A$ is empty and $\hat{\sigma} \leq (v - c_R)/(v - c_A)$, there exists an equilibrium $\{\mathbb{X}, \theta, \sigma, \lambda, \bar{V}_A, \bar{V}_R\}$ with $\mathbb{X} = \{x_P = 0\}$ and $\sigma(x_P) = \hat{\sigma}$. Moreover, the equilibrium $\bar{V}_R$ and $\bar{V}_A$ are unique if $\hat{\sigma} < (v - c_R)/(v - c_A)$.*

If $\hat{\sigma} = (v - c_R)/(v - c_A)$, the uniqueness result does not apply because $\bar{V}_A$ is pinned down by the (possibly mixed) bidding strategy of buyers on platform x_P .

Proposition 3.5 (Semi-Pooling Equilibrium). *If $\mathbb{X}_A$ is empty and $\hat{\sigma} > (v - c_R)/(v - c_A)$, there exists an equilibrium $\{\mathbb{X}, \theta, \sigma, \lambda, \bar{V}_A, \bar{V}_R\}$ with $\mathbb{X} = \{x_P = 0, x_A = x_A^*\}$, $\sigma(x_P) = (v - c_R)/(v - c_A)$ and $\sigma(x_A) = 1$. Moreover, the equilibrium $\bar{V}_R$ and $\bar{V}_A$ are unique.*

Propositions 3.4 and 3.5 point to a relationship between the aggregate composition of sellers, $\hat{\sigma}$, and the instance of pooling or semi-pooling in equilibrium. When most sellers are relaxed so that $\hat{\sigma} \leq (v - c_R)/(v - c_A)$, the appeal of the bilateral surplus is enough to discourage costly signalling among anxious sellers. When the aggregate share of anxious sellers is high and $\hat{\sigma} > (v - c_R)/(v - c_A)$, buyers' bilateral bidding strategy under pooling involves offers of c_A , which does not leave any surplus for anxious sellers. Anxious sellers face an incentive to signal their type by listing on a costly marketing platform. With $\mathbb{X}_A$ empty, however, separation cannot be an equilibrium outcome since there does not exist a pair of incentive compatible platforms. According to Proposition 3.5, the equilibrium necessarily involves semi-pooling, whereby the mixed strategy of buyers when making offers in a bilateral match begets indifference among anxious sellers between platforms $x_P = 0$ and $x_A = x_A^*$. Mixing between offers c_R and c_A is an optimal strategy for buyers, according to Lemma 1, as long as the composition of sellers listing on x_P is exactly $\sigma(x_P) = (v - c_R)/(v - c_A)$.

The bordered regions in Figure 2 designate the type of equilibrium (separating, pooling, or semi-pooling) that exists for different parameter values. (The shaded region is related to market efficiency and can be ignored for now.) One intuitive insight from the figure is that the pooling region expands both vertically and to the right as the difference in the valuations of relaxed and anxious sellers, $c_R - c_A$, becomes smaller. In other words, there is a larger parameter space that gives rise to a pooling equilibrium when sellers are relatively homogeneous.

3.2 Efficiency Implications of Costly Signalling

There is no guarantee that market separation by means of costly marketing platforms improves efficiency relative to the case without signalling. This section studies the implications for net social surplus of the introduction of a costly signalling technology. In an environment

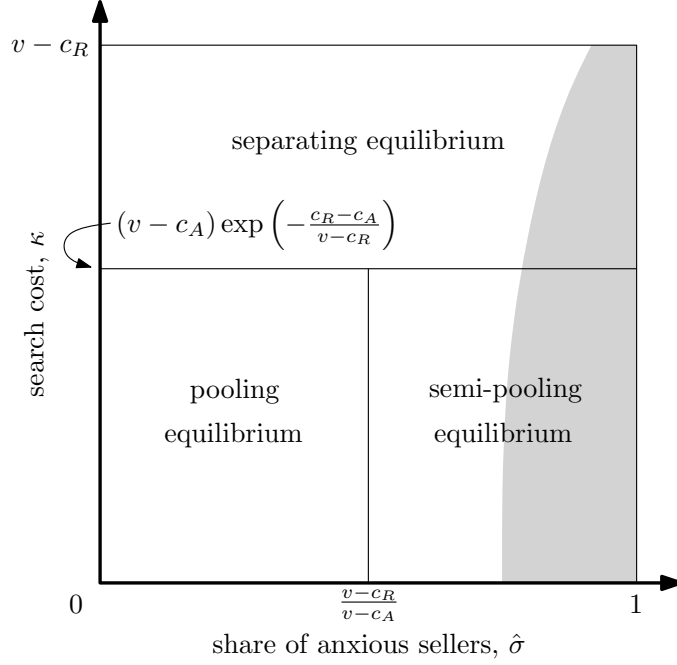


Figure 2: Graphic characterization of the equilibrium with costly marketing platforms.

with private information but without costly signalling, the average net surplus per house is

$$\hat{\sigma}V_A(\hat{\sigma}, \theta_P) + (1 - \hat{\sigma})V_R(\theta_P), \quad (11)$$

where θ_P satisfies the free entry condition,

$$U(\hat{\sigma}, \theta_P) = e^{-\theta_P} \max \{ \hat{\sigma}(v - c_A), v - c_R \} = \kappa. \quad (12)$$

When the parameters are such that the equilibrium with costly marketing platforms is one of pooling, the costly signalling technology is left idle and the equilibrium allocation is unchanged. Under separation or semi-pooling, on the other hand, the average net surplus

per house with costly signalling is equal to

$$\hat{\sigma} [V_A(1, \theta_A) - x_A^*] + (1 - \hat{\sigma})V_R(\theta_R), \quad (13)$$

where θ_A and θ_R satisfy

$$e^{-\theta_A}(v - c_A) = e^{-\theta_R}(v - c_R) = \kappa. \quad (14)$$

When platform x_A^* is active, anxious sellers benefit from higher market tightness, but if $\hat{\sigma}(v - c_A) > v - c_R$, relaxed sellers participate in a submarket with a lower buyer-seller ratio than they would without market separation. The following proposition determines the necessary and sufficient conditions for net social surplus to decrease with the introduction of listing fees as an endogenous signalling mechanism.

Proposition 3.6. *The accessibility of the costly marketing platforms strictly reduces net social surplus relative to the equilibrium without costly signalling if and only if $\hat{\sigma}$ and κ satisfy*

$$(1 - \hat{\sigma})(v - c_R) - \hat{\sigma}^2(c_R - c_A) < 0, \text{ and} \quad (15)$$

$$\kappa < \exp \left(1 + \frac{[(1 - \hat{\sigma})(v - c_R) + \hat{\sigma}(v - c_A)] \log(\hat{\sigma}(v - c_A)) - \hat{\sigma}^2(c_R - c_A) \log(v - c_A) - \hat{\sigma}(v - c_A) \log(v - c_R)}{(1 - \hat{\sigma})(v - c_R) - \hat{\sigma}^2(c_R - c_A)} \right). \quad (16)$$

While (15) and (16) are difficult to interpret, they are in fact the parameter restrictions required for the surplus in (11) to exceed the surplus in (13) after imposing buyers' free entry conditions, (12) and (14), and using the binding incentive compatibility constraint for relaxed sellers in (10) to replace x_A^* . Satisfying these conditions requires a large share of anxious sellers and inexpensive entry (*i.e.*, high $\hat{\sigma}$ and low κ). In such cases, the pooling allocation closely resembles the costly platform for anxious sellers: market tightness is high, and buyers make low offers of c_A in the event of a bilateral match. With a large enough share of anxious sellers, allocating sufficient resources to the costly signalling technology to achieve separation outweighs the benefit of improved liquidity. The parameter space for which costly signalling reduces net social surplus is indicated by the shaded region in Figure 2. In the

unshaded area, costly signalling improves net social surplus, with strict market efficiency gains in the unshaded parameter space that involves market separation or semi-pooling.

4 An Application to Housing Markets

In the context of housing markets, the theoretical predictions are consistent with the notion that sellers signal their willingness to sell via their expenditures on real estate marketing services. Anxious sellers might be willing to spend more on a real estate agent, for example, if it allows them to distinguish themselves from relaxed sellers and attract additional potential buyers. A positive listing fee or commission could, in some circumstances, provide an incentive compatible means of separating the market by seller type: anxious sellers choose to be represented by high-fee agents and as a result attract more buyers, while relaxed sellers are more likely to list their house without the assistance of an agent, or with limited-service discount realtors. Even if real estate agents provide no technological advantage in the matching process, the theory predicts that incentive compatible listing fees can segment the market to alleviate the information problem and increase market activity as long as housing is otherwise sufficiently illiquid.

This provides a theoretical foundation for the empirical results of Hendel et al. (2009). They compare housing market transactions on two different marketing platforms: the MLS and the newly established low-cost FSBO Madison. They find that after controlling for observable house characteristics, transaction prices are similar between the two platforms, but that homes listed with a traditional real estate broker have shorter times on the market and are more likely to ultimately result in a transaction. They also find evidence of endogenous sorting and report that impatient sellers are more likely to list with the high-commission, high-service option. Levitt and Syverson (2008a) similarly compare limited-service and full-service real estate agents, while Bernheim and Meer (2008) study Stanford Housing listings with and without the representation of a real estate broker. Both find that time on the

market is shorter among sellers that choose the more costly method for listing their home.¹³ These findings are consistent with the liquidity or informational role of costly marketing platforms in frictional housing markets.

4.1 Fixed Rate Commissions

The theoretical analysis thus far assumed flat, upfront listings fees. In practice, a fixed rate commission structure is more common: real estate contracts in North America typically specify a commission of 5 to 7 percent of the sale price (Hsieh and Moretti, 2003; Federal Trade Commission and U.S. Department of Justice, 2007). From a principal-agent perspective,¹⁴ a fee that increases with the sale price is more likely to induce effort on the part of the real estate agent, whereas upfront non-refundable fees are least effective at motivating the agent. While I abstract from effective marketing expenditures and principal-agent matters in this paper, it is important to check the robustness of the results when listing contracts are modelled to reflect the type of contract commonly observed between a home seller and her agent.

Modifying the analysis to accommodate fixed percentage fees is accomplished by replacing the upfront non-refundable listing fee, x , with a *listing agreement* that specifies a commission as a fixed share $z \in [0, 1]$ of the sale price. Adjusting the fee structure in this manner introduces an additional effect that further hinders market separation. A buyer now has to

¹³While the empirical result that sales prices are indistinguishable between the two marketing platforms in Hendel et al. (2009), Levitt and Syverson (2008a), and Bernheim and Meer (2008) seemingly contradicts the prediction of the model, it is important to note that prices include sales commissions. Moreover, straightforward model extensions, such as *effective* marketing services provided by real estate brokers to improve the expected quality of a match by influencing the buyer's valuation, can generate similar average precommission prices across platforms in a separating equilibrium despite distinct transaction price distributions.

¹⁴Many theoretical models of real estate agents focus on the principal-agent relationship between the seller (the principal) and the realtor (the agent) (Yavaş, 1992; Yavaş and Yang, 1995). The attention of empiricists has also been aimed at the principal-agent problem in the market for real estate services. Levitt and Syverson (2008b) and Rutherford et al. (2005) find evidence to support the hypothesis that sellers' and their agents' incentives are misaligned by comparing the selling prices and duration on the market in transactions when the real estate agent is a third party and when the agent is also the owner of the home.

increase his take-it-or-leave-it offer in a bilateral match until the seller deems it acceptable after fees are deducted. More specifically, when the commission rate is z and the seller is willing to accept c , an acceptable offer must be at least $c/(1 - z)$. This reduces the payoff to a buyer in a bilateral match and implies that buyer entry is affected by the commission rate. Higher fees result in fewer buyers, which offsets the desired effect of attracting more buyers with a costly signal. While this effect works against incentive compatibility and market separation, the analysis is not fundamentally altered when fixed rate fees are imposed: it merely implies that a smaller region of the parameter space generates a separating equilibrium.

Listing agreements typically specify a list price. What if the agent's commission can be made contingent on procuring a "ready, willing, and able" buyer (i.e., contingent on receiving an offer at or above the list price)? This form of contract is often observed in North American real estate markets.¹⁵ Even if the seller rejects an offer equal to or above the list price, it is considered that the real estate agent has provided the agreed upon service and the seller must still pay the commission. The "ready, willing, and able" clause (hereinafter, the RWA clause) is useful (but not essential) for achieving market separating in equilibrium.

Proposition 4.1. *Adding a list price and RWA clause to the listing agreement tightens the incentive compatibility constraint for relaxed sellers.*

This structure of real estate listing agreement dissuades patient sellers from mimicking impatient ones and entering the market with higher demand. By introducing a penalty for rejecting a take-it-or-leave-it offer in a bilateral match, it becomes easier for anxious sellers to hire real estate agents as a costly means of signalling their motivation to sell.

¹⁵For example, a listing agreement with the Toronto Real Estate Board stipulates that "the Seller agrees to pay the Listing Brokerage a commission of% of the sale price of the Property or for any valid offer to purchase the Property from any source whatsoever obtained during the Listing Period and on the terms and conditions set out in this Agreement."

5 Concluding Remarks

In this paper, I present a model of a decentralized market with search and information frictions. Sellers with unobservable reservation values compete to attract buyers but are unable to commit to an asking price. The transaction price is determined by the equilibrium bidding/acceptance strategies of buyers/sellers and depends on the number of buyers in a match. Private information and the absence of commitment leads to an inefficient equilibrium; some buyer-seller matches fail to realize gains from trade because buyers are uninformed when they submit their offer, and inefficient buyer entry can further diminish market activity. By allowing sellers to sort among seemingly irrelevant but costly marketing platforms, there is a potential for endogenous market separation to alleviate the information problem and improve market efficiency.

The model can qualitatively account for observed realtor facts in housing markets. For instance, many home sellers choose to enlist the services of a costly real estate agent, despite evidence suggesting that the value of hiring an agent, measured in terms of transaction prices and time on the market, is not enough to justify high commission rates between five and seven percent. With seller heterogeneity and private information, the theory sheds light on the demand for real estate marketing services. Realtors provide marketing and advertising services that are *observable* (e.g., yard signs, billboards, newspaper advertisements, and online listings), and can structure their listing contracts in a way that affords a potential solution to an information problem. A motivated seller can select a high-cost real estate agent as a means of signalling a high willingness to sell, thus attracting more buyers. According to the theory, the demand for costly marketing services can, in some circumstances, persist even if one endorses the view that the services provided are ineffective.

A Omitted Proofs

Proof of Proposition 2.1. To show that buyer entry is optimal, denote by Π_A the social surplus from putting a house on the market when the seller has reservation value c_A . As long as one or more potential buyers show up, the surplus is $v - c_A$.

$$\Pi_A(\theta) = e^{-\theta} \sum_{k=1}^{\infty} \frac{\theta^k}{k!} (v - c_A) = (1 - e^{-\theta})(v - c_A) \quad (\text{A.1})$$

Define Π_R in the analogous manner for houses available for purchase from relaxed sellers. Taking the numbers of sellers as given, the planner has only to choose the number of buyers visiting sellers of each type to maximize total social surplus less entry costs. Equivalently, the social planner can choose θ_A and θ_R to maximize the average social surplus per house.

$$\max_{\theta_A, \theta_R} \hat{\sigma} [\Pi_A(\theta_A) - \kappa\theta_A] + (1 - \hat{\sigma}) [\Pi_R(\theta_R) - \kappa\theta_R] \quad (\text{A.2})$$

After substituting for Π_A using the definition in equation (A.1) and likewise for Π_R , the first order conditions for the planner's problem are

$$e^{-\theta_A}(v - c_A) = \kappa \quad (\text{A.3})$$

$$e^{-\theta_R}(v - c_R) = \kappa \quad (\text{A.4})$$

These are the same equations as the free entry conditions for buyers in the market with full information, equations (4) and (5).

If the planner is further constrained by private information, assigning different buyer-seller ratios to different types of sellers would require inducing sellers to reveal their type. It is straightforward to check that the above allocation is implementable using the pricing protocol described in Section 2 but with public and binding reserve prices equal to the reservation values. $\square$

Proof of Proposition 2.2. With observable non-binding list prices, the market can potentially be characterized by multiple submarkets. Consider one submarket for sellers with list price p_1 , and another submarket for sellers with list price p_2 . Assume WLOG that $\sigma_1 \geq \sigma_2$.

The buyer-seller ratios are determined by the free entry conditions, $U(\sigma_1, \theta_1) = U(\sigma_2, \theta_2) = \kappa$. Given $\sigma_1 \geq \sigma_2$ and the properties of U (part (i) of Lemma 2), this implies $\theta_1 \geq \theta_2$: relatively more buyers enter the submarket with the highest share of anxious sellers.

The incentive compatibility of $\sigma_1 \geq \sigma_2$ requires relaxed sellers to (weakly) prefer submarket 2: $V_R(\theta_2) \geq V_R(\theta_1)$. By the properties of V_R (part (ii) of Lemma 2), this requires $\theta_2 \geq \theta_1$: relaxed sellers choose an appropriate list price to participate in the submarket with the highest buyer-seller ratio.

Combining the implications of free entry ($\theta_1 \geq \theta_2$) and of the pricing decisions of relaxed sellers ($\theta_2 \geq \theta_1$) yields the result that $\theta_1 = \theta_2$. $\square$

Proof of Proposition 3.1. The incentive compatibility constraint for relaxed sellers, (8), is

$$\begin{aligned} V_R(\theta_R) &= [1 - (1 + \theta_R)e^{-\theta_R}](v - c_R) \\ &\geq [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - x_A = V_R(\theta_A) - x_A \end{aligned}$$

After substitutions using the payoff functions for anxious sellers, this constraint can be written

$$V_A(0, \theta_R) - [1 - e^{-\theta_R}](c_R - c_A) \geq V_A(1, \theta_A) - x_A - [1 - (1 + \theta_A)e^{-\theta_A}](c_R - c_A)$$

or

$$[e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}](c_R - c_A) \geq V_A(1, \theta_A) - x_A - V_A(0, \theta_R) \quad (\text{A.5})$$

The incentive compatibility constraint for anxious sellers, (7), requires

$$V_A(1, \theta_A) - x_A - V_A(0, \theta_R) \geq 0 \quad (\text{A.6})$$

There exists an x_A satisfying (A.5) and (A.6) if and only if

$$[e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}](c_R - c_A) \geq 0 \quad \Longleftrightarrow \quad e^{\theta_A - \theta_R} \geq 1 + \theta_A \quad (\text{A.7})$$

The free entry conditions determine θ_A and θ_R :

$$\begin{aligned} U(0, \theta_R) = e^{-\theta_R}(v - c_R) = \kappa & \Longleftrightarrow \theta_R = \log(v - c_R) - \log \kappa \\ U(1, \theta_A) = e^{-\theta_A}(v - c_A) = \kappa & \Longleftrightarrow \theta_A = \log(v - c_A) - \log \kappa \end{aligned}$$

With these expressions for θ_A and θ_R , inequality (A.7) reduces to condition (9). $\square$

Proof of Proposition 3.2(i). Part 3 of Definition 3.1 implies

$$U(\sigma(x) = 0, \theta(x)) = e^{-\theta(x)}(v - c_R) = \kappa$$

which establishes that $\theta(x) = \theta_R$. Suppose FSOC that $x > 0$, and consider platform $x' < x$. Part 3 of Definition 3.1 requires

$$U(\sigma(x'), \theta(x')) = e^{-\theta(x')} \max \{ \sigma(x')(v - c_A), v - c_R \} \leq \kappa$$

which implies $\theta(x') \geq \theta(x)$. Moreover,

$$V_R(\theta(x')) - x' > V_R(\theta(x')) - x \geq V_R(\theta(x)) - x = \bar{V}_R$$

where the first inequality follows from the construction of x' , the second relies on $\theta(x') \geq \theta(x)$

and part (ii) of Lemma 2, and the equality uses part 2 of Definition 3.1. The result that sellers can attract more buyers at a lower-cost platform and achieve $V_R(\theta(x')) - x' > \bar{V}_R$ is a contradiction since it violates part 2 of Definition 3.1. $\square$

Proof of Proposition 3.2(ii). Suppose FSOC that $\sigma(x) \in ((v - c_R)/(v - c_A), 1)$. Part 3 of Definition 3.1 implies

$$U(\sigma(x) > (v - c_R)/(v - c_A), \theta(x)) = e^{-\theta(x)} \sigma(x)(v - c_A) = \kappa \quad (\text{A.8})$$

Consider platform $x' > x$. Part 3 of Definition 3.1 requires

$$U(\sigma(x'), \theta(x')) = e^{-\theta(x')} \max \{ \sigma(x')(v - c_A), v - c_R \} \leq \kappa \quad (\text{A.9})$$

and part 1 of Definition 3.1 requires

$$V_A(\sigma(x'), \theta(x')) - x' \leq V_A(\sigma(x) > (v - c_R)/(v - c_A), \theta(x)) - x = \bar{V}_A \quad (\text{A.10})$$

If $\sigma(x') = 1$, (A.8) and (A.9) imply $\theta(x) < \theta(x')$ (strict because $\sigma(x) < 1$). If instead $\sigma(x') < 1$, part 2 of Definition 3.1 requires either $\theta(x') = \infty$ or the following equality:

$$V_R(\theta(x')) - x' = V_R(\theta(x)) - x = \bar{V}_R$$

so that $\theta(x) < \theta(x')$ obtains in all cases. Therefore, using part (iii) of Lemma 2,

$$V_A(\sigma(x'), \theta(x')) > V_A(\sigma(x) > (v - c_R)/(v - c_A), \theta(x))$$

Since $x' > x$ was chosen arbitrarily, there exists an x' close enough to x to violate (A.10): a contradiction. This establishes that $\sigma(x) \leq (v - c_R)/(v - c_A)$. Proving that $x = 0$ and $\theta(x) = \theta_R$ uses the same arguments as the proof of Proposition 3.2(i). $\square$

Proof of Proposition 3.2(iii). Part 3 of Definition 3.1 implies

$$U(\sigma(x) = 1, \theta(x)) = e^{-\theta(x)}(v - c_A) = \kappa$$

which establishes that $\theta(x) = \theta_A$.

By part 4 of Definition 3.1, there exists $x' \in \mathbb{X}$ with $\sigma(x') < 1$. By parts (i) and (ii) of Proposition 3.2, $x' = 0$, $\sigma(x') \leq (v - c_R)/(v - c_A)$, and $\theta(x') = \theta_R$. Parts 1 and 2 of Definition 3.1 thus require

$$\bar{V}_A = V_A(\sigma(x) = 1, \theta(x) = \theta_A) - x \geq V_A(\sigma(x') < (v - c_R)/(v - c_A), \theta(x') = \theta_R) \quad (\text{A.11})$$

and

$$\bar{V}_R = V_R(\theta(x') = \theta_R) \geq V_R(\theta(x) = \theta_A) - x \quad (\text{A.12})$$

Since x satisfies (A.11) and (A.12), it must also satisfy constraints (7) and (8). The set $\mathbb{X}_A$ is therefore non-empty.

Suppose FSOc that $x \neq x_A^* \equiv V_R(\theta_A) - V_R(\theta_R)$. Any $x < x_A^*$ would violate (A.12). If $x > x_A^*$, consider platform x_A^* . If $\sigma(x_A^*) = 1$, part 3 of Definition 3.1 requires

$$U(\sigma(x_A^*) = 1, \theta(x_A^*)) = e^{-\theta(x_A^*)}(v - c_A) \leq e^{-\theta_A}(v - c_A) = U(\sigma(x) = 1, \theta(x) = \theta_A) = \kappa$$

which means $\theta(x_A^*) \geq \theta_A$. If instead $\sigma(x_A^*) < 1$, either $\theta(x_A^*) = \infty$ or $\theta(x_A^*) = \theta_A$ by part 2 of Definition 3.1 and, in the latter case, by the definition of x_A^* . So $\theta(x_A^*) \geq \theta_A$ in all cases and therefore, using $x_A^* < x$ and Lemma 2,

$$V_A(\sigma(x_A^*), \theta(x_A^*) \geq \theta_A) - x_A^* > V_A(\sigma(x) = 1, \theta(x) = \theta_A) - x = \bar{V}_A$$

which violates part 1 of Definition 3.1: a contradiction. □

Definition A.1. For $x \notin \mathbb{X}$, set $\theta(x)$ to satisfy

$$\begin{aligned}\bar{V}_A &= V_A(\sigma(x) = 1, \theta(x)) - x & \text{if } \frac{\bar{V}_A + x}{v - c_A} < \frac{\bar{V}_R + x}{v - c_R} \\ \bar{V}_R &= V_R(\theta(x)) - x & \text{if } \frac{\bar{V}_A + x}{v - c_A} \geq \frac{\bar{V}_R + x}{v - c_R}\end{aligned}$$

or $\theta(x) = \infty$ if there is no solution to the relevant equation. Set

$$\sigma(x) = \begin{cases} 0 & \text{if } \frac{\bar{V}_R + x}{v - c_R} \leq \frac{\bar{V}_A + x - \theta(x)e^{-\theta(x)}(c_R - c_A)}{v - c_A} < \frac{\bar{V}_A + x}{v - c_A} \\ \frac{v - c_R}{v - c_A} & \text{if } \frac{\bar{V}_A + x - \theta(x)e^{-\theta(x)}(c_R - c_A)}{v - c_A} < \frac{\bar{V}_R + x}{v - c_R} \leq \frac{\bar{V}_A + x}{v - c_A} \\ 1 & \text{if } \frac{\bar{V}_A + x}{v - c_A} < \frac{\bar{V}_R + x}{v - c_R} \end{cases}$$

When $\sigma(x) = (v - c_R)/(v - c_A)$, set $\pi(x)$ to satisfy

$$\frac{\bar{V}_A + x - \pi(x)\theta(x)e^{-\theta(x)}(c_R - c_A)}{v - c_A} = \frac{\bar{V}_R + x}{v - c_R}$$

Proof of Proposition 3.3. Proof of existence by construction: $\mathbb{X} = \{x_R = 0, x_A = x_A^*\}$; θ satisfies $\theta(x_R) = \theta_R$ and $\theta(x_A) = \theta_A$; σ satisfies $\sigma(x_R) = 0$ and $\sigma(x_A) = 1$; λ is such that and $\lambda(x_R) = (1 - \hat{\sigma})S\theta_R$ and $\lambda(x_A) = \hat{\sigma}S\theta_A$; and $\{\bar{V}_R, \bar{V}_A\}$ are given by

$$\begin{aligned}\bar{V}_R &= [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) \\ \bar{V}_A &= [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - x_A\end{aligned}$$

For $x \notin \mathbb{X}$, set $\theta(x)$, $\sigma(x)$ and $\pi(x)$ according to Definition A.1.

Part 1 of Definition 3.1: By construction, θ , σ and π guarantee that $\bar{V}_A \geq V_A(\sigma(x), \theta(x)) - x$ for all $x \in \mathbb{R}_+$, with equality if $\theta(x) < \infty$ and $\sigma(x) > 0$. Moreover, $\bar{V}_A = V_A(\sigma(x), \theta(x)) - x$ for platform $x = x_A \in \mathbb{X}$.

Part 2 of Definition 3.1: By construction, θ and σ guarantee that $\bar{V}_R \geq V_R(\theta(x)) - x$ for all $x \in \mathbb{R}_+$, with equality if $\theta(x) < \infty$ and $\sigma(x) < 1$. Moreover, $\bar{V}_R = V_R(\theta(x)) - x$ for

platform $x = x_R \in \mathbb{X}$.

Part 3 of Definition 3.1: $\theta(x_R) = \theta_R$ and $\theta(x_A) = \theta_A$ imply that buyers' optimal entry/search holds for platforms x_R and x_A . Suppose FSOC that there exists some other platform $x \notin \mathbb{X}$ such that $U(\sigma(x), \theta(x)) > \kappa$. This requires either $\sigma(x) = 1$ and $\theta(x) < \theta_A$ or $\sigma(x) \leq (v - c_R)/(v - c_A)$ and $\theta(x) < \theta_R$. If $\sigma(x) \leq (v - c_R)/(v - c_A)$, by construction either $\theta(x) = \infty$ or $\theta(x)$ satisfies $\bar{V}_R = V_R(\theta(x)) - x$. Therefore $\theta(x) < \theta_R$ would require $x < 0$: a contradiction. If instead $\sigma(x) = 1$, by construction either $\theta(x) = \infty$ or $\theta(x)$ satisfies $\bar{V}_A = V_A(\sigma(x) = 1, \theta(x)) - x$. Therefore $\theta(x) < \theta_A$ would require $x < x_A$. The expressions for $\bar{V}_R$ and $\bar{V}_A$, along with $x_A = x_A^*$ imply

$$\frac{\bar{V}_R + x_A}{v - c_R} = \frac{V_R(\theta_A)}{v - c_R} = [1 - (1 + \theta_A)e^{-\theta_A}] = \frac{\bar{V}_A + x_A}{v - c_A}$$

and $x < x_A$ further implies

$$\frac{\bar{V}_R + x}{v - c_R} < \frac{\bar{V}_A + x}{v - c_A}$$

This is a contradiction given the construction of σ and θ according to Definition A.1.

Part 4 of Definition 3.1: Market clearing holds given the way λ is constructed.

Since all equilibrium conditions of Definition 3.1 are satisfied, this completes the proof of existence.

To prove uniqueness, note that parts (i) and (ii) of Proposition 3.2 show that in any equilibrium, $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) < 1$ implies $x = 0$ and $\theta(x) = \theta_R$. Part 2 of Definition 3.1 therefore guarantees that

$$\bar{V}_R = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R)$$

is the unique value for relaxed sellers. Parts (ii) and (iii) of Proposition 3.2 show that in any equilibrium, $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) > 0$ implies either $x = x_A^*$, $\sigma(x) = 1$ and $\theta(x) = \theta_A$ or $x = 0$, $\sigma(x) \leq (v - c_R)/(v - c_A)$ and $\theta(x) = \theta_R$. In the first case, part 1 of

Definition 3.1 implies that

$$\bar{V}_A = [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - x_A^* \quad (\text{A.13})$$

is the value for anxious sellers. In the latter case,

$$\bar{V}_A = V_A(\sigma(x) \leq (v - c_R)/(v - c_A), \theta(x) = \theta_R) \leq [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - x_A^* \quad (\text{A.14})$$

where the inequality uses the fact that $x_A^* \in \mathbb{X}_A$. A strict inequality in (A.14) would violate part 1 of Definition 3.1 for platform x_A^* ,¹⁶ which establishes that (A.13) is the unique value for anxious sellers when $\mathbb{X}_A$ is non-empty. $\square$

Proof of Proposition 3.4. Proof of existence by construction: $\mathbb{X} = \{x_P = 0\}$; θ satisfies $\theta(x_P) = \theta_R$; σ satisfies $\sigma(x_P) = \hat{\sigma}$; λ is such that $\lambda(x_P) = S\theta_R$; and $\{\bar{V}_R, \bar{V}_A\}$ are given by

$$\begin{aligned} \bar{V}_R &= [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) \\ \bar{V}_A &= [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_A) + \theta_R e^{-\theta_R} (c_R - c_A) \end{aligned}$$

or, if $\hat{\sigma} = (v - c_R)/(v - c_A)$, $\bar{V}_A$ and $\pi(x_P) = \pi_R$ satisfy

$$\bar{V}_A = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_A) + \pi_R \theta_R e^{-\theta_R} (c_R - c_A)$$

¹⁶Suppose FSOC that the inequality is strict, and consider platform x_A^* . If $\sigma(x_A^*) = 1$, part 3 of Definition 3.1 would imply $\theta(x_A^*) \geq \theta_A$. If instead $\sigma(x_A^*) < 1$, part 2 of Definition 3.1 would require either $\theta(x_A^*) = \infty$ or

$$\bar{V}_R = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) = [1 - (1 + \theta(x_A^*))e^{-\theta(x_A^*)}] (v - c_R) - x_A^*$$

which would mean $\theta(x_A^*) = \theta_A$ by the construction of x_A^* . In all cases, $\theta(x_A^*) \geq \theta_A$. Hence,

$$V_A(\sigma(x_A^*), \theta(x_A^*) \geq \theta_A) - x_A^* \geq [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - x_A^* > \bar{V}_A$$

and the assumed strict inequality contradicts part 1 of Definition 3.1.

and, to ensure that $\bar{V}_A > V_A(1, \theta_A) - x_A^*$,

$$\pi_R > \frac{[(1 + \theta_R)e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}]}{\theta_R e^{-\theta_R}}.$$

For $x \notin \mathbb{X}$, set $\theta(x)$, $\sigma(x)$ and $\pi(x)$ according to Definition A.1.

Part 1 of Definition 3.1: By construction, θ , σ and π guarantee that $\bar{V}_A \geq V_A(\sigma(x), \theta(x)) - x$ for all $x \in \mathbb{R}_+$, with equality if $\theta(x) < \infty$ and $\sigma(x) > 0$. Moreover, $\bar{V}_A = V_A(\sigma(x), \theta(x)) - x$ for platform $x = x_P \in \mathbb{X}$.

Part 2 of Definition 3.1: By construction, θ and σ guarantee that $\bar{V}_R \geq V_R(\theta(x)) - x$ for all $x \in \mathbb{R}_+$, with equality if $\theta(x) < \infty$ and $\sigma(x) < 1$. Moreover, $\bar{V}_R = V_R(\theta(x)) - x$ for platform $x = x_P \in \mathbb{X}$.

Part 3 of Definition 3.1: $\theta(x_P) = \theta_R$ implies that buyers' optimal entry/search holds for platform x_P . Suppose FSOc that there exists some other platform $x \notin \mathbb{X}$ such that $U(\sigma(x), \theta(x)) > \kappa$. This requires either $\sigma(x) = 1$ and $\theta(x) < \theta_A$ or $\sigma(x) \leq (v - c_R)/(v - c_A)$ and $\theta(x) < \theta_R$. If $\sigma(x) \leq (v - c_R)/(v - c_A)$, by construction either $\theta(x) = \infty$ or $\theta(x)$ satisfies $\bar{V}_R = V_R(\theta(x)) - x$. Therefore $\theta(x) < \theta_R$ would require $x < 0$: a contradiction. If instead $\sigma(x) = 1$, by construction either $\theta(x) = \infty$ or $\theta(x)$ satisfies

$$\bar{V}_A = V_A(\sigma(x) = 1, \theta(x)) - x \tag{A.15}$$

Moreover,

$$\bar{V}_A > V_A(\sigma_A = 1, \theta_A) - x_A^* > V_A(\sigma(x) = 1, \theta(x)) - x_A^* \tag{A.16}$$

where the first inequality is an implication of $\mathbb{X}_A = \emptyset$, and the second inequality uses the assumption that $\theta(x) < \theta_A$. (A.15) and (A.16) imply $x < x_A^*$, while (A.16) and the definition of x_A^* imply

$$\frac{\bar{V}_R + x_A^*}{v - c_R} < \frac{\bar{V}_A + x_A^*}{v - c_A}$$

Since $x < x_A^*$, this also implies

$$\frac{\bar{V}_R + x}{v - c_R} < \frac{\bar{V}_A + x}{v - c_A}$$

This is a contradiction given the construction of σ and θ according to Definition A.1.

Part 4 of Definition 3.1: Market clearing holds given the way λ is constructed.

Since all of equilibrium conditions of Definition 3.1 are satisfied, this completes the proof of existence.

To prove uniqueness, note that parts (i) and (ii) of Proposition 3.2 show that in any equilibrium, $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) < 1$ implies $x = 0$ and $\theta(x) = \theta_R$. Part 2 of Definition 3.1 therefore guarantees that

$$\bar{V}_R = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R)$$

is the unique value for relaxed sellers. Parts (ii) and (iii) of Proposition 3.2 show that in any equilibrium, $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) > 0$ implies either $x = x_A^*$, $\sigma(x) = 1$ and $\theta(x) = \theta_A$ or $x = 0$, $\sigma(x) \leq (v - c_R)/(v - c_A)$ and $\theta(x) = \theta_R$. With $\hat{\sigma} < (v - c_R)/(v - c_A)$, part 4 of Definition 3.1 and $\mathbb{X}_A = \emptyset$ preclude the first case. This leaves

$$\bar{V}_A = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_A) + \theta_R e^{-\theta_R} (c_R - c_A)$$

as the unique value for anxious sellers when $\mathbb{X}_A = \emptyset$ and $\hat{\sigma} < (v - c_R)/(v - c_A)$. $\square$

Proof of Proposition 3.5. Proof of existence by construction: $\mathbb{X} = \{x_P = 0, x_A = x_A^*\}$; θ satisfies $\theta(x_P) = \theta_R$ and $\theta(x_A) = \theta_A$; σ satisfies $\sigma(x_P) = (v - c_R)/(v - c_A)$ and $\sigma(x_A) = 1$; λ is such that

$$\begin{aligned} \lambda(x_P) &= \frac{(1 - \hat{\sigma})(v - c_A)}{c_R - c_A} S\theta_R \\ \lambda(x_A) &= \frac{\hat{\sigma}(v - c_A) - (v - c_R)}{c_R - c_A} S\theta_A \end{aligned}$$

$\{\bar{V}_R, \bar{V}_A\}$ are given by

$$\begin{aligned}\bar{V}_R &= [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) \\ \bar{V}_A &= [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - x_A\end{aligned}$$

and $\pi(x_P) = \pi_R$ satisfies

$$\bar{V}_A = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_A) + \pi_R \theta_R e^{-\theta_R} (c_R - c_A)$$

For $x \notin \mathbb{X}$, set $\theta(x)$, $\sigma(x)$ and $\pi(x)$ according to Definition A.1.

Part 1 of Definition 3.1: By construction, θ , σ and π guarantee that $\bar{V}_A \geq V_A(\sigma(x), \theta(x)) - x$ for all $x \in \mathbb{R}_+$, with equality if $\theta(x) < \infty$ and $\sigma(x) > 0$. Moreover, $\bar{V}_A = V_A(\sigma(x), \theta(x)) - x$ for platforms $x = x_A, x_P \in \mathbb{X}$.

Part 2 of Definition 3.1: By construction, θ and σ guarantee that $\bar{V}_R \geq V_R(\theta(x)) - x$ for all $x \in \mathbb{R}_+$, with equality if $\theta(x) < \infty$ and $\sigma(x) < 1$. Moreover, $\bar{V}_R = V_R(\theta(x)) - x$ for platform $x = x_P \in \mathbb{X}$.

Part 3 of Definition 3.1: $\theta(x_P) = \theta_R$ and $\theta(x_A) = \theta_A$ imply that buyers' optimal entry/search holds for platforms x_P and x_A . For inactive platforms, the proof is the same as the relevant part of the proof of Proposition 3.3.

Part 4 of Definition 3.1: Market clearing holds given the way λ is constructed.

Since all of equilibrium conditions of Definition 3.1 are satisfied, this completes the proof of existence.

To prove uniqueness, note that parts (i) and (ii) of Proposition 3.2 show that in any equilibrium, $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) < 1$ implies $x = 0$ and $\theta(x) = \theta_R$. Part 2 of Definition 3.1 therefore guarantees that

$$\bar{V}_R = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R)$$

is the unique value for relaxed sellers. Parts (ii) and (iii) of Proposition 3.2 show that in any equilibrium, $x \in \mathbb{X}$ with $\theta(x) < \infty$ and $\sigma(x) > 0$ implies either $x = x_A^*$, $\sigma(x) = 1$ and $\theta(x) = \theta_A$ or $x = 0$, $\sigma(x) \leq (v - c_R)/(v - c_A)$ and $\theta(x) = \theta_R$. With $\hat{\sigma} > (v - c_A)/(v - c_R)$, part 4 of Definition 3.1 require an active platform with $\sigma(x) = 1$. Part (iii) of Proposition 3.2 and part 1 of Definition 3.1 then guarantee that

$$\bar{V}_A = [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - x_A^*$$

is the unique value for anxious sellers when $\mathbb{X}_A = \emptyset$ and $\hat{\sigma} > (v - c_R)/(v - c_A)$. $\square$

Proof of Proposition 3.6. First consider the case where $\hat{\sigma} \leq (v - c_R)/(v - c_A)$. Optimal buyer entry/search implies $\theta_P = \theta_R$. Relaxed sellers are therefore indifferent about the signalling technology, while anxious sellers make use of platform x_A^* if it beneficial to do so (*i.e.*, if $\mathbb{X}_A$ is non-empty or, equivalently, if the parameters satisfy condition (9) in Proposition 3.1). The accessibility of the costly signalling technology therefore improves net social surplus relative to the equilibrium without costly signalling when $\hat{\sigma} \leq (v - c_R)/(v - c_A)$.

Next consider the case where $\hat{\sigma} > (v - c_R)/(v - c_A)$. Using the expressions for V_R and V_A given by (2) and (3) and the definition of x_A^* from (10), the surplus in an equilibrium without costly signalling in (11) exceeds the surplus with costly marketing platforms in (13) if and only if

$$\hat{\sigma} [(1 + \theta_P)e^{-\theta_P} - (1 - \theta_A)e^{-\theta_A}] (c_R - c_A) < [(1 + \theta_R)e^{-\theta_R} - (1 + \theta_P)e^{-\theta_P}] (v - c_R)$$

Substituting for θ_P , θ_A and θ_R using the free entry conditions in (12) and (14) yields

$$\begin{aligned} & [(1 - \hat{\sigma})(v - c_R) - \hat{\sigma}^2(c_R - c_A)] (\log \kappa - 1) \\ & > [(1 - \hat{\sigma})(v - c_R) + \hat{\sigma}(v - c_A)] \log(\hat{\sigma}(v - c_A)) \\ & \quad - \hat{\sigma}^2(c_R - c_A) \log(v - c_A) - \hat{\sigma}(v - c_A) \log(v - c_R) \end{aligned}$$

Rearranging to isolate κ , this inequality is equivalent to

$$\begin{aligned}\kappa &> \tilde{\kappa}(\hat{\sigma}) \quad \text{if } \frac{v-c_R}{v-c_A} < \hat{\sigma} < \tilde{\sigma} \\ \kappa &< \tilde{\kappa}(\hat{\sigma}) \quad \text{if } \tilde{\sigma} \leq \hat{\sigma} < 1\end{aligned}$$

where $\tilde{\sigma}$ satisfies $\tilde{\sigma}^2(c_R - c_A) = (1 - \tilde{\sigma})(v - c_R)$ and $(v - c_R)/(v - c_A) < \tilde{\sigma} < 1$, and

$$\tilde{\kappa}(\hat{\sigma}) = \exp \left(1 + \frac{[(1-\hat{\sigma})(v-c_R)+\hat{\sigma}(v-c_A)] \log(\hat{\sigma}(v-c_A)) - \hat{\sigma}^2(c_R-c_A) \log(v-c_A) - \hat{\sigma}(v-c_A) \log(v-c_R)}{(1-\hat{\sigma})(v-c_R) - \hat{\sigma}^2(c_R-c_A)} \right)$$

When $(v - c_R)/(v - c_A) < \hat{\sigma} < \tilde{\sigma}$, the inequality can never be satisfied for acceptable parameters because it would imply an entry cost that prohibits buyer entry:

$$\kappa > \tilde{\kappa}(\hat{\sigma}) > \hat{\sigma}(v - c_A) > (v - c_R)$$

When $\hat{\sigma} \geq \tilde{\sigma}$, model parameters that satisfy $\kappa < \tilde{\kappa}(\hat{\sigma})$ guarantee that a pooling allocation achieves a higher surplus than the equilibrium with costly marketing platforms. $\square$

Proof of Proposition 4.1. Consider a listing agreement designed for type A sellers with list price $p_A = c_A/(1 - z_A)$. The RWA clause in this case has no effect on buyers' bidding strategies in a type A submarket, and does not impact anxious sellers' expected payoff. Type R sellers, on the other hand, now face a penalty equal to $\min \{z_A c_A/(1 - z_A), c_R - c_A\}$ in a bilateral match in the type A submarket.¹⁷

The binding type R incentive compatibility constraint in the absence of a RWA clause determines the listing fee, z_A^* :

$$[1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) = [1 - (1 + \theta_A)e^{-\theta_A}] [(1 - z_A^*)v - c_R] \quad (\text{A.17})$$

¹⁷The minimization operator reflects the mimicking seller's choice between rejecting the offer but paying the agent's commission, $z_A c_A/(1 - z_A)$, and going ahead with the transaction at a price below her reservation value.

With the RWA clause, this same condition becomes

$$\begin{aligned} & [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R) \\ & > [1 - (1 + \theta_A)e^{-\theta_A}] [(1 - z_A^*)v - c_R] - \theta_A e^{-\theta_A} \min \left\{ \frac{z_A^* c_A}{1 - z_A^*}, c_R - c_A \right\} \end{aligned} \quad (\text{A.18})$$

The inequality is no longer binding because of the extra term on the right hand side. Commission rates less than z_A^* are now incentive compatible for relaxed sellers. $\square$

B Idiosyncratic Match Quality

The purpose of this appendix is to show that *ex post* heterogeneity on the demand side of the market does not generate endogenous market separation in the version of the model without costly signalling. Rather than buyers with common values as in Section 2, suppose that upon visiting a seller, the value that a buyer assigns to owning the home is a match specific random variable to reflect the idiosyncratic quality of the match. The subtle differences between units that are only observable by visiting and inspecting a house result in variation in buyers' *ex post* valuations. Assume the match specific valuation, v , is known only to the buyer, and is an independent draw from a standard uniform distribution.

If a buyer meets a seller and a transaction takes place at price p , the payoff to the buyer is $v - p$, and the payoff to the seller is $p - c$, where $v \in [0, 1]$ refers to the quality of the match between the buyer and the house, and $c \in \{c_A, c_R\}$ refers to the reservation value of the seller. For convenience, normalize $c_A = 0$ and $c_R = c \in (0, 1)$.

B.1 Buyers' Bidding Strategies

Consider a market characterized by the buyer-seller ratio θ , and the fraction of motivated sellers σ . Let $b_k(v)$ denote buyers' symmetric, increasing, and differentiable bidding strategy in a match with k other buyers. In equilibrium, it is optimal for a bidder with value v to

submit $b = b_k(v)$ if all k other buyers visiting the same seller submit bids according to b_k . Bidding b' in a match with k other buyers yields expected payoff

$$(b_k^{-1}(b'))^k \times \begin{cases} \sigma(v - b') & \text{if } b' < c \\ v - b' & \text{if } b' \geq c \end{cases}$$

Maximizing with respect to b' yields a first-order condition. To impose symmetric equilibrium bidding behaviour, the first-order condition is evaluated at $b' = b_k(v)$, which yields

$$\frac{d}{dv} b_k(v) + \frac{k b_k(v)}{v} = k$$

Since $b_k(0) = 0$,

$$b_k(v) = \frac{k}{k+1} v, \quad v \in [0, \hat{v})$$

where $\hat{v}$ satisfies

$$\sigma \left(\hat{v} - \frac{k}{k+1} \hat{v} \right) = \hat{v} - c \quad \Rightarrow \quad \hat{v} = \left(\frac{k+1}{k+1-\sigma} \right) c$$

which follows from the other boundary condition, $b_k(\hat{v}) = c$. The bidding function is therefore

$$b_k(v) = \begin{cases} \frac{k}{k+1} v & \text{if } v \in \left[0, \frac{(k+1)c}{k+1-\sigma} \right) \\ \frac{k}{k+1} v + \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{(k+1-\sigma)v} \right]^{k+1} v & \text{if } v \in \left[\frac{(k+1)c}{k+1-\sigma}, 1 \right] \end{cases} \quad (\text{B.1})$$

B.2 Expected Payoffs and Free Entry

The expected payoff to a buyer with value v in a match with k other buyers is

$$u(v, \sigma, k) = \begin{cases} \frac{\sigma v^{k+1}}{k+1} & \text{if } v \in \left[0, \frac{(k+1)c}{k+1-\sigma} \right) \\ \frac{v^{k+1}}{k+1} - \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+1} & \text{if } v \in \left[\frac{(k+1)c}{k+1-\sigma}, 1 \right] \end{cases} \quad (\text{B.2})$$

For a buyer that has visited a seller and observed the number of other bidders, but has yet to inspect the house and draw a value, the expected payoff is

$$\begin{aligned} u(\sigma, k) &= \int_0^{\frac{(k+1)c}{k+1-\sigma}} \frac{\sigma v^{k+1}}{k+1} dv + \int_{\frac{(k+1)c}{k+1-\sigma}}^1 \left(\frac{v^{k+1}}{k+1} - \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+1} \right) dv \\ &= \frac{1}{(k+2)(k+1)} + \frac{1-\sigma}{k+2} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+2} - \frac{1-\sigma}{k+1} \left[\frac{(k+1)c}{k+1-\sigma} \right]^{k+1} \end{aligned} \quad (\text{B.3})$$

The buyer's *ex ante* expected payoff is

$$U(\sigma, \theta) = e^{-\theta} \sum_{k=0}^{\infty} \frac{\theta^k}{k!} u(\sigma, k) \quad (\text{B.4})$$

For example, if $\sigma = 1$ and $\theta = \theta_A$ (a market with only anxious sellers), the expected payoff to a buyer is

$$U(1, \theta_A) = e^{-\theta_A} \sum_{k=0}^{\infty} \frac{\theta_A^k}{(k+2)!} = \frac{1 - (1 + \theta_A)e^{-\theta_A}}{\theta_A^2} \quad (\text{B.5})$$

and if $\sigma = 0$ and $\theta = \theta_R$ (a market with only relaxed sellers), the expected payoff is

$$\begin{aligned} U(0, \theta_R) &= e^{-\theta_R} \sum_{k=0}^{\infty} \frac{\theta_R^k}{(k+2)!} [1 + (k+1)c^{k+2} - (k+2)c^{k+1}] \\ &= \frac{1 - [1 + \theta_R(1-c)] e^{-\theta_R(1-c)}}{\theta_R^2} \end{aligned} \quad (\text{B.6})$$

A relaxed seller's expected payoff when matched with k buyers is

$$\begin{aligned} v_R(\sigma, k) &= \int_{\frac{kc}{k-\sigma}}^1 (k)v^{k-1}(b_{k-1}(v) - c)dv \\ &= \frac{k-1}{k+1} + \frac{c}{k+1} \left(\frac{kc}{k-\sigma} \right)^k + (1-\sigma) \left(\frac{kc}{k-\sigma} \right)^k \left[1 - \frac{k}{k+1} \left(\frac{kc}{k-\sigma} \right) \right] - c \end{aligned} \quad (\text{B.7})$$

which is the expectation of the highest bid less the reservation value, conditional on the

highest bid exceeding c (*i.e.*, conditional on the highest valuation exceeding $kc/(k - \sigma)$). Conditioning on a high enough bid reflects the absence of commitment; every offer below c is costlessly rejected by a relaxed seller.

Prior to buyer arrival, the expected payoff takes into account the matching probabilities:

$$V_R(\sigma, \theta) = e^{-\theta} \sum_{k=2}^{\infty} \frac{\theta^k}{k!} v_R(\sigma, k) \quad (\text{B.8})$$

In a market with only relaxed sellers, the seller's expected payoff is

$$\begin{aligned} V_R(0, \theta_R) &= e^{-\theta_R} \sum_{k=2}^{\infty} \frac{\theta_R^k}{k!} \left(\frac{k-1}{k+1} [1 - c^{k+1}] + c^k - c \right) \\ &= (1 - c) \left[1 + e^{-\theta_R(1-c)} \right] - \frac{2}{\theta_R} \left[1 - e^{-\theta_R(1-c)} \right] \end{aligned} \quad (\text{B.9})$$

whereas deviating to a market with only anxious sellers yields an expected payoff equal to

$$\begin{aligned} V_R(1, \theta_A) &= e^{-\theta_A} \sum_{k=2}^{\infty} \frac{\theta_A^k}{k!} \left(\frac{k-1}{k+1} + \left(\frac{kc}{k-1} \right)^k \frac{c}{k+1} - c \right) \\ &= 1 + e^{-\theta_A} - \frac{2}{\theta_A} [1 - e^{-\theta_A}] - c [1 - (1 + \theta_A)e^{-\theta_A}] \\ &\quad + \frac{e^{-\theta_A}}{\theta_A} \sum_{k=3}^{\infty} \frac{(c\theta_A)^k}{k!} \left(\frac{k-1}{k-2} \right)^{k-1} \end{aligned} \quad (\text{B.10})$$

B.3 The Non-Existence of a Separating Equilibrium

I demonstrate numerically the non-existence of a separating equilibrium in the version of the model without costly marketing platforms. For a grid of values for $c \in (0, 1)$, I first use the free entry conditions, $U(1, \theta_A) = \kappa$ and $U(0, \theta_R) = \kappa$ to check that both submarkets are active (*i.e.*, to verify that buyers find it worthwhile to enter both markets) and to calculate the buyer-seller ratios, θ_A and θ_R . If κ is low enough for both markets to attract buyers, I compute $V_R(0, \theta_R)$ and $V_R(1, \theta_A)$ using (B.9) and (B.10). I then show that $V_R(1, \theta_A) >$

$V_R(0, \theta_R)$, which violates incentive compatibility of market separation. I repeat this exercise for different values of κ . Figures 3 and 4 illustrate the incentive for relaxed sellers to deviate from a market with only relaxed sellers to a market with only anxious sellers.

The intuition for relaxed sellers' incentive to deviate from market separation is similar to the model without *ex post* buyer heterogeneity (see Proposition 2.2). The free entry conditions imply $\theta_A > \theta_R$. The market for anxious sellers therefore has higher matching probabilities and more intense competition among bidders. It is the appeal of higher offers without requiring commitment to sell when only low offers are received that precludes incentive compatible market separation.

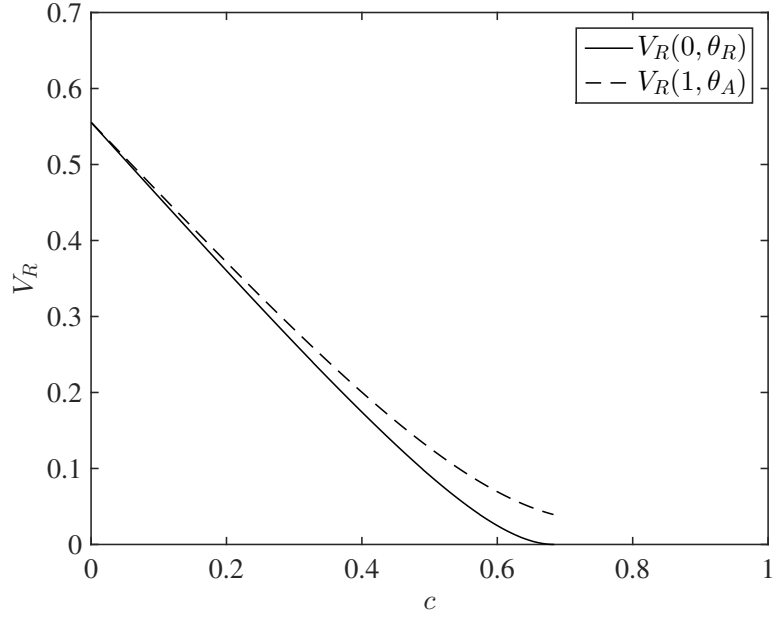


Figure 3: A relaxed seller's expected payoff with market separation ($\kappa = .05$).

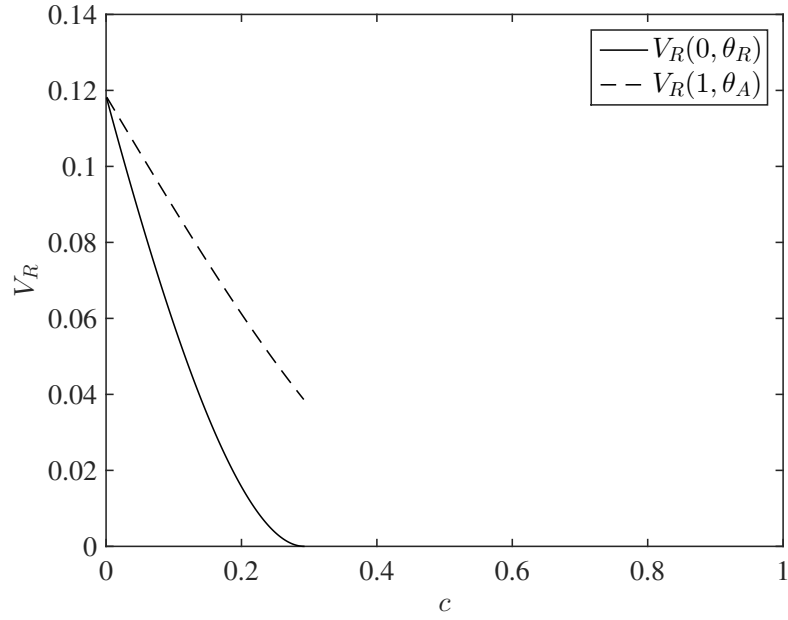


Figure 4: A relaxed seller's expected payoff with market separation ($\kappa = .25$).

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