# Safe haven currencies: A dependence-switching copula approach *

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July 2024

**Abstract**

This paper presents a unique approach to investigating the safe haven properties of five major currencies: the US dollar, the Japanese yen, the Swiss franc, the euro, and the British pound. Unlike other studies, we employ a flexible dependence-switching copula model to examine the joint tail dependence between these currencies and global market risk. This innovative method allows us to measure the strength of safe haven currencies directly. Using daily data from January 1999 to June 2024, our empirical findings reveal the US dollar's continued status as a safe haven currency during periods of heightened global risk aversion. The yen also maintains its safe haven attributes, even in the presence of the US dollar's safe haven behaviour. The Swiss franc exhibits safe haven characteristics, albeit less pronounced than the US dollar. In contrast, the euro and the pound demonstrate the weakest safe haven characteristics among the five currencies.

Keywords: Safe haven currency; dependence-switching copula; VIX.

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* **Acknowledgments:** Cathy Ning is grateful to Eric Zivot and Yanqin Fan at the University of Washington for their discussions, constructive comments, and suggestions during her sabbatical visit. We thank the participants at the 58th Annual Meeting of the Canadian Economics Association, the Economics Brownbag seminar series at Toronto Metropolitan University in 2021–2022, and the 2024 Econometrics and Macroeconomics Seminar Series at the University of Washington for their valuable feedback on earlier versions of the paper.

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# 1 Introduction

A safe haven currency is expected to maintain or increase its value during periods of major economic, financial or geopolitical shocks with cross-national effects, such as the 2007–2008 financial crisis or the COVID-19 pandemic. Such shocks have recently increased interest in studying safe haven currencies among economists, private investors, and policymakers seeking to navigate volatile market conditions, manage risk, and preserve capital in an uncertain global economic environment. This paper investigates the safe haven properties of five major currencies: the US dollar, USD, the Japanese yen, JPY, the Swiss franc, CHF, the euro, EUR, and the British pound, GBP, in the context of a flexible dependence-switching copula model. This model allows for both positive and negative joint tail dependence between exchange rates relative to the US dollar and measures of global volatility or global market risk.

The five currencies investigated in this paper are commonly studied in the safe haven literature. The seigniorage of the USD can partially justify its status as a safe haven (Prasad, 2015). It serves as a crucial vehicle currency for clearing international payments and invoicing trade flows. Some countries have even outright adopted the USD as their own or linked their currencies at a fixed exchange rate (Calvo, 2002). Also, the USD largely appreciated during the global financial crisis of 2007–2008. McCauley and McGuire (2009) attribute the USD's appreciation during this crisis mainly to dollar shortages resulting from a surge in dollar funding costs, flights to US Treasury bills, and the write-down of USD-denominated assets that led to over-hedged books. There is a consensus that the Japanese yen is a safe haven. One possible explanation is that foreign ownership of Japanese debt is very low, resulting in less selling pressure during times of crisis (Hoque, 2012). Other contributing factors include a chronic trade surplus and deflation. It is worth noting that recent actions by the Bank of Japan to increase its balance sheet could potentially undermine the yen's status as a safe haven.

The euro and the pound sterling are often included in studies related to safe haven currencies (Ranaldo and Söderlind, 2010; Coudert et al., 2014; Hossfeld and MacDonald, 2015; Fatum and Yamamoto, 2016; Tachibana, 2018; Wong and Fong, 2018). According to data provided by the Bank for International Settlements (BIS), the euro and the pound rank as the second and fourth most traded currencies, respectively. This fact partly motivates their inclusion in studies examining currencies that serve as safe harbours of value. Other considerations include their international status, reputation, and underlying fundamentals (De Santis, 2012; Habib and Stracca, 2012; Coudert et al., 2014; Hossfeld and MacDonald, 2015). Contrary to the traditional view, the CHF's performance as a safe haven is not as straightforward as it seems. While it does provide hedging benefits on average (Kugler and Weder, 2004), the CHF's performance is mixed (Grisse and Nitschka, 2015; Hossfeld and MacDonald, 2015; Fatum and Yamamoto, 2016). This inconsistency in its performance is an intriguing aspect that warrants further exploration. Coudert et al. (2014) argue that the franc's long-run appreciation is more of a continuous trend than a specific reaction to global financial turmoil. Switzerland's perceived relative stability can be attributed to its intrinsic low-risk profile, which includes protecting individual financial rights and adhering to a foreign policy of neutrality.

In the safe haven literature, the most widely used measure of global volatility is the Chicago Board Options Exchange (CBOE) Volatility Index or the VIX index (see (Ranaldo and Söderlind, 2010; Coudert et al., 2014; De Bock and de Carvalho Filho, 2015; Fatum and Yamamoto, 2016)). Earlier studies have established that the VIX is sufficient to measure market-wide distress (Collin-Dufresn et al., 2001; Pan and Singleton, 2008; Gyntelberg and Schrimpf, 2011; Rey, 2015). Other studies measuring market risk via the VIX include Carr and Wu (2006), Whaley (2009), Gonzalez-Perez (2015), and Martin (2017). Habib and Stracca (2012) use a panel approach and the VIX to measure global risk to determine which fundamentals are essential to safe haven behaviour. Although they advocate using alternative risk estimates to check the robustness of the results, they find that the results are similar regardless. Similarly, Fatum and Yamamoto's (2016) primary market uncertain benchmark is also the VIX, and they establish the causal relationship to be from the VIX to the exchange rate. Brunnermeier et al. (2008) document that currency crashes are positively correlated with increases in the VIX, and their alternative measure to the VIX gives a similar result but with less statistical power. Wong and Fong (2018) construct a risk aversion index using the first principal component from nine stock market volatility indexes. Similar to Habib and Stracca (2012) and Fatum and Yamamoto (2016), there is a high positive correlation between these indexes. Moreover, their results are consistent with studies mainly using the VIX (for example, Grisse and Nitschka (2015)).

In the literature, the study of safe haven currencies often relies on regression analysis using the sign/magnitude of regression coefficients on market risk, such as VIX, see Ranaldo and Söderlind (2010), Coudert et al. (2014), Hossfeld and MacDonald (2015), Fatum and Yamamoto (2016), and Wong and Fong (2018). For instance, Ranaldo and Söderlind (2010) and Coudert et al. (2014) use risk factor models and a smooth transition regression (STR) model, respectively. Lee (2017) employs Markov regime-switching vector autoregressive (MS-VAR) models to assess the negative relationship between safe haven currencies and risky assets. Reboredo (2013) uses tail dependence and the copula method to assess the role of gold as a safe haven or hedge against the USD. Tachibana (2018) adopts a copula-based approach to characterize the relationship between stock returns and exchange rate changes to identify safe haven and hedge currencies.

In the present paper, we investigate safe haven currencies based on the tail dependence between each of four exchange rates relative to the US dollar and the VIX. For instance, a positive tail correlation between an exchange rate and the VIX signifies that the US dollar serves as a safe haven currency. Specifically, we evaluate the tail dependence between an exchange rate and the VIX by employing a dependence-switching copula model (Wang et al., 2013, 2018). In addition to the VIX, we also use volatility indexes based on the European and Swiss markets as robustness checks.

Our dependence-switching copula (DSC) approach differs from the existing literature in three important aspects. First, tail dependence allows us to measure directly how currency values change under extreme market conditions, especially during market downturns when the risk is elevated, which is directly tied to the concept of safe haven currencies. Unlike the linear regression approach commonly used in the literature (e.g., Ranaldo and Söderlind (2010), Hossfeld and MacDonald (2015), Fatum and Yamamoto (2016)), which does not capture the currency value changes specifically during the time of elevated risk, the DSC model captures the safe haven properties of a currency precisely when it is most relevant.

Second, the model allows for both positive and negative tail dependence and any potential switches between them. This is useful, as positive and negative dependence corresponds to the safe haven status of the denominator (US dollar) and the numerator currencies, respectively, when the market risk is heightened. It can also measure the relative strength of the safe haven property. Thus, we do not impose directional restrictions on their dependence and hence do not restrict the safe haven behaviour of any currency. The regression type of method commonly used in the literature and the copula-based models used in the literature (see Reboredo (2013) and Tachibana (2018)) do not allow for the dependence direction to change and hence restrict the safe haven behaviour.

Third, the DSC approach yields new empirical insights. Using the DSC model and estimating the tail dependence between exchange rates and VIX, we find that the USD emerges as a strong, safe haven currency.[1] Further, we find that the JPY also serves as a safe haven currency, which is not dominated by the appreciation of the USD in times of heightened global risk aversion. However, while the EUR and the GBP exhibit safe haven characteristics, these effects are overshadowed by the similar behavior of the USD. The CHF also exhibits safe haven features to a lesser extent than the USD and the JPY but more so than the EUR and GBP. Our results are generally consistent with previous studies by Ranaldo and Söderlind (2010), Hossfeld and MacDonald (2015), Fatum and Yamamoto (2016), and Wong and Fong (2018). However, our research advances the literature by offering a more direct method for identifying safe haven currencies and providing a deeper understanding of safe haven currencies. For instance, while Wong and Fong's (2018) study offers valuable insights, it does not quantify the degree or relative strength of safe haven characteristics across different currencies to the extent that our research does.

In summary, we find that the USD, the JPY, and the CHF exhibit safe haven characteristics at varying levels. The USD emerges as a strong, safe haven currency relative to all other currencies except the JPY, which stands out as the strongest safe haven currency. The GBP and the EUR show the weakest safe haven characteristics.

The remainder of the paper is structured as follows. Section 2 provides the details of the joint model, the dependence measures, the marginal models, and the estimation procedure. Section 3 describes the data and provides the summary statistics. Our empirical results are presented in Section 4. Section 5 concludes.

[1] Our research also uncovers evidence that the USD may be playing an influential role as a carry funding currency, a finding that sheds new light on its influence in the market. For instance, Hossfeld and MacDonald (2015) provide compelling data to support this claim.

# 2 Models and estimation

## 2.1 Joint model: the dependence-switching copula model

Copulas ($C(\cdot, ..., \cdot)$) are functions that combine univariate marginal distributions ($F_1(\cdot), ..., F_N(\cdot)$) to construct their respective joint distribution ($F(\cdot, ..., \cdot)$) (Sklar, 1959). To measure the dependence structure between the market risk and currencies, we adopt the dependence-switching copula model of Wang et al. (2018). Let $u_{1,t}$ and $u_{2,t}$ be the probability integral transforms that pertain to the volatility index, the VIX and the exchange rate percentage changes, with $C(\cdot, \cdot)$ representing the copula function that describes the dependence structure between the two series. Consider the following state-varying copula:

$$C\big(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_1^C, \boldsymbol{\theta}_0^C | S_t\big) = \begin{cases} C_1\big(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_1^C\big), & \text{if } S_t = 1 \\ C_0\big(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_0^C\big), & \text{if } S_t = 0 \end{cases}, \tag{1}$$

where $S_t$ is an unobserved state variable representing either a positive ($S_t = 1$) or a negative ($S_t = 0$) dependence state.

In the above equation, $C_1\big(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_1^C\big)$ and $C_0\big(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_0^C\big)$ are two mixed copulas corresponding to the positive and negative dependence states or regimes, with $\boldsymbol{\theta}_1^C$ and $\boldsymbol{\theta}_0^C$ denoting the corresponding parameter vectors for each state, respectively. The state variable $S_t$ follows a Markov chain with a transition matrix $\mathbf{\Pi}$, i.e.,

$$\mathbf{\Pi} = \begin{bmatrix} \Pi_{00} & 1 - \Pi_{00} \\ 1 - \Pi_{11} & \Pi_{11} \end{bmatrix}, \tag{2}$$

where $\Pi_{00} = \Pr(S_t = 0|S_{t-1} = 0)$ and $\Pi_{11} = \Pr(S_t = 1|S_{t-1} = 1)$. Hence, $\Pi_{00}$ is the probability of two successive negative dependence states, with $1 - \Pi_{00}$ quantifying the chance of transitioning out of the negative dependence state (regime). A similar interpretation will hold for $\Pi_{11}$ and $1-\Pi_{11}$.

Since the mixture of two Archimedean copulae is also an Archimedean copula (see Nelsen (2006)), we mix a Clayton copula ($C^C(\cdot,\cdot)$) with a survival Clayton copula ($C^{SC}(\cdot,\cdot)$) to allow for both left and right tail dependence:

$$C_1(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_1^C) = w_1 \times C^C(u_{1,t}, u_{2,t}; \chi_1) + (1 - w_1) \times C^{SC}(u_{1,t}, u_{2,t}; \chi_2), \tag{3}$$

$$C_0(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_0^C) = w_2 \times C^C(1 - u_{1,t}, u_{2,t}; \chi_3) + (1 - w_2) \times C^{SC}(1 - u_{1,t}, u_{2,t}; \chi_4), \tag{4}$$

where $\boldsymbol{\theta}_1^C = (\chi_1, \chi_2, w_1)'$, $\boldsymbol{\theta}_0^C = (\chi_3, \chi_4, w_2)'$, and $\chi_k \in (0, \infty)$ $(k = 1, 2, 3, 4)$ are copula parameters, and $w_1$ and $w_2$ are the weights for the corresponding copulas. The bivariate Clayton copula and the survival Clayton copula are given by $C^C(u_1, u_2; \chi_k) = (u_1^{-\chi_k} + u_2^{-\chi_k} - 1)^{-\frac{1}{\chi_k}}$ and $C^{SC}(u_1, u_2; \chi_k) = u_1 + u_2 - 1 + C^C(1 - u_1, 1 - u_2; \chi_k)$. The advantage of this joint model is that it allows for both positive and negative dependence states or regimes with the capacity to transition between them. For a more complete discussion on the properties of copulas, please refer to Nelsen (2006) and Joe (1997).

## 2.2 Dependence measures

We can assess dependence using a set of measures derived from the joint model, including the correlation coefficient and tail dependence. Standard dependence measures include the Pearson correlation, Spearman's $\rho$ and Kendall's $\tau$. Rank correlations such as Spearman's $\rho$ and Kendall's $\tau$ are often preferred over the Pearson correlation coefficient because they can capture nonlinearities, are invariant under increasing transformations, and depend only on the joint distribution (Joe, 2014; McNeil et al., 2015). Additionally, Kendall's $\tau$, which represents the difference between the probability of concordance and discordance for two random variables, can be estimated through the copula parameter ($\chi_k$) as $\tau_k(X_1, X_2) = \frac{\chi_k}{2+\chi_k}$. The correlation coefficient ($\rho_k$) can then be computed using Kendall's $\tau$, i.e., $\rho_k(X_1, X_2) = \sin\left(\frac{\pi}{2} \times \tau_k(X_1, X_2)\right)$. From a risk management perspective, in addition to the commonly used dependence measures mentioned above, tail dependence is critical in decision-making during extreme market conditions (Embrechts et al., 2002).[2]

To capture the dependence at extremes, we use tail dependence measures. By definition, the upper (lower) or right (left) tail dependence measure quantifies the probability of observing a high (low) $U_1$, given that $U_2$ is high (low). The Clayton copula only exhibits lower tail dependence, while the survival Clayton only has upper tail dependence. The mixture of the two copulas, in combination with the state-switching aspect, results in four different configurations of tail dependence. In the positive dependence regime, we have $\lambda_1^{LL} = w_1 \times 2^{-\frac{1}{\chi_1}}$ and $\lambda_2^{RR} = (1-w_1) \times 2^{-\frac{1}{\chi_2}}$, where $\lambda_1^{LL}$ and $\lambda_2^{RR}$ are the left (lower) and right (upper) tail dependence coefficients, respectively. They measure the dependence when both variables are at the lower or upper end of the spectrum, respectively. $\lambda_2^{RR}$ is crucial for identifying safe haven currencies as it corresponds to situations where market risk is exceptionally high and the base currency appreciates.[3]

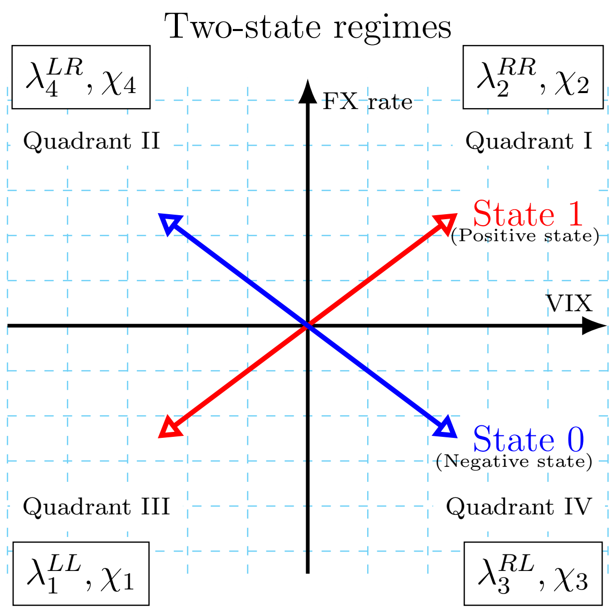
In the negative dependence regime, we have $\lambda_3^{RL} = w_2 \times 2^{-\frac{1}{\chi_3}}$ and $\lambda_4^{LR} = (1 - w_2) \times 2^{-\frac{1}{\chi_4}}$, where $\lambda_3^{RL}$ measures the dependence when volatility (market risk) is overly high while the currency significantly depreciates in value, indicating a non-safe haven currency. We estimate these tail dependence parameters by employing the dependence-switching copula model described above.[4]

[2]Often, there could be a large discrepancy between the nonparametric estimates of the above dependence measures and their counterparts derived from a specific model (Rodriguez, 2007; Garcia and Tsafack, 2011).

[3]The tail dependence coefficient $\lambda_1^{LL}$ can be related to short carry trade positions on the USD.

[4]Traditional nonparametric approaches to estimating tail dependence, as in Caillault and Guegan (2005) and

Figure 1: Schematic of dependence regimes




*Notes*: The coefficient of tail dependence is denoted by $\lambda_k^{\cdot\cdot}$. The right and left tails of the distribution are denoted by $R$ and $L$. The parameter $\lambda_2^{RR} = (1 - w_1) \times 2^{-\frac{1}{\chi_2}}$ is associated with the safe haven behaviour of the USD and $\lambda_3^{RL} = w_2 \times 2^{-\frac{1}{\chi_3}}$ with that of the other currencies. When the VIX is low, the tail dependence is estimated with $\lambda_1^{LL} = w_1 \times 2^{-\frac{1}{\chi_1}}$ and $\lambda_4^{LR} = (1 - w_2) \times 2^{-\frac{1}{\chi_4}}$.

Figure 1 summarizes the dependence structure of the regime-switching copula model. The first and fourth quadrants of Figure 1 are particularly crucial for identifying safe haven currencies, as they represent extremely high VIX levels with heightened market risk. The exchange rate is expressed as the value of the quote currency per unit of the base currency. In the first quadrant, when the VIX is high, the base currency appreciates, indicating its safe haven status. Conversely, in the fourth

Schmidt and Stadtmüller (2006), will suffer from the curse of dimensionality and the limited number of extreme data points (Aas, 2004; Garcia and Tsafack, 2011).

quadrant, the quote currency appreciates when the VIX is high, indicative of the safe haven status of the quote currency.

### 2.3 Marginal model and the empirical CDF

To remove possible serial correlation and heteroscedasticity from the data in order to obtain the independent and identically distributed (i.i.d.) inputs for the copulas, we specify an AR(1)-GARCH(1,1) model with the Generalized Error Distribution (GED) for the marginal model. Let $x_{1,t}$ and $x_{2,t}$ denote the measure of market risk and the log difference of the foreign exchange (FX) rate variable. The mean process follows an AR(1) for both series as

$$x_{i,t} = \mu_i + \phi_i x_{i,t-1} + \epsilon_{i,t};\ \epsilon_{i,t}|I_{i,t-1} \sim e;\ i = 1, 2, \tag{5}$$

where $I_{i,t-1}$ is the information available at time $t-1$ for $i$ and $\mu_i$ is the time-invariant intercept. $\phi_i$ is a coefficient.

The GARCH(1,1) process for the conditional variance of $\epsilon_{i,t}$ is

$$h_{i,t} = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i h_{i,t-1};\ i = 1, 2, \tag{6}$$

where $\omega_i$ is the intercept and $h_{i,t}$ is the variance of $\epsilon_{i,t}|I_{i,t-1}$. The coefficients $\alpha_i$ and $\beta_i$ correspond to the ARCH and the GARCH terms, respectively. We use the GED for the innovations to capture the heavy tails in the data (see Nelson (1991)). The GED density is as follows:

$$f(z|\nu) = \varkappa(\nu) \times \exp\big[-2^{-1}|z \times \varrho_\nu^{-1}|^\nu\big],\ \ -\infty < z < \infty,\ \nu > 0, \tag{7}$$

where $\nu$ is a tail-thickness parameter and $\varkappa(\nu)$ and $\varrho_\nu$ are given by

$$\varrho_\nu = [2^{-\frac{2}{\nu}}\Gamma(\nu^{-1})[\Gamma(3\nu^{-1})]^{-1}]^{\frac{1}{2}} \text{ and } \varkappa(\nu) = \nu[\varrho_\nu \times 2^{1+v^{-1}}\Gamma(\nu^{-1})]^{-1}, \tag{8}$$

where $\Gamma(\cdot)$ is a gamma function. The GED can be transformed into the skewed version based on the transformation of Fernández and Steel (1998). We denote the skewness parameter with $\zeta$ and the parameter vector of the marginal model with $\boldsymbol{\theta}_i^M = (\mu_i, ..., \nu_i)'$.[5] We utilize the robust standard errors according to the quasi-maximum likelihood (QML) method in the context of Bollerslev and Wooldridge (1992).

To avoid misidentification, following literature (see Chen and Fan (2006)), the margins of the standardized residuals $F_1$ and $F_2$ are estimated nonparametrically by the empirical cumulative distribution function (ECDF) of the standardized residuals $z_{i,t}$, obtaining uniformly distributed $u_{i,t}= F_i(z_{i,t};\boldsymbol{\theta}_i^M|I_{i,t-1})$ as follows:

$$\hat{F}_i(z_{i,t};\boldsymbol{\theta}_i^M|I_{i,t-1}) = \frac{1}{T+1}\sum_{n=1}^{T}\mathbf{1}(z_{i,n} \leq z_{i,t}), \tag{9}$$

for $i = 1, 2$, where $\mathbf{1}(\cdot)$ is an indicator function, which takes the value 1 when its argument is true and 0 otherwise.

[5] The interpretation of the skewness parameter $\zeta$ depends on the fact that $\zeta^2$ is equal to the ratio of probability masses above and below the mode of the distribution.

## 2.4 Estimation

The density of the dependence-switching copula model can be expressed as follows

$$f(x_{1,t}, x_{2,t}; \boldsymbol{\theta}^M, \boldsymbol{\theta}^C, \boldsymbol{\Pi}|I_{t-1}) = \left(\sum_{j=0}^{1} \Pr(S_t = j|I_{t-1}; \boldsymbol{\Theta}) \times c_j(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_j^C, \boldsymbol{\Pi}|I_{t-1})\right)\left(\prod_{i=1}^{2} f_i(x_{i,t}; \boldsymbol{\theta}_i^M|I_{i,t-1})\right), \tag{10}$$

where $\boldsymbol{\theta}^C = (\boldsymbol{\theta}_1^C, \boldsymbol{\theta}_0^C)$, $\boldsymbol{\theta}^M = (\boldsymbol{\theta}_1^M, \boldsymbol{\theta}_2^M)$, and $\boldsymbol{\Theta} = (\boldsymbol{\theta}^M, \boldsymbol{\theta}^C, \boldsymbol{\Pi})$. The function $c_j$ is the copula density function under regime $j$ with parameter set $\boldsymbol{\theta}_j^C$. The log-likelihood function of (10) is

$$\begin{aligned} L(\boldsymbol{\Theta}; I_T) &= L_C(u_{1,t}, u_{2,t}, \boldsymbol{\theta}^C, \boldsymbol{\Pi}; I_T) + \sum_{i=1}^{2} L_i(\boldsymbol{\theta}_i^M; I_{i,T}), \\ &= L_C(u_{1,t}, u_{2,t}, \boldsymbol{\theta}^C, \boldsymbol{\Pi}; I_T) + \sum_{i=1}^{2}\sum_{t=1}^{T} \log(f_i(x_{i,t}; \boldsymbol{\theta}_i^M|I_{i,t-1})), \end{aligned} \tag{11}$$

where $L_C$ and $L_i$ are the log of the copula density and the marginal density, respectively. We can obtain the unconditional copula density by integrating as follows:

$$L_C(u_{1,t}, u_{2,t}, \boldsymbol{\theta}^C, \boldsymbol{\Pi}; I_T) = \sum_{t=1}^{T}\sum_{j=0}^{1}[\log[\Pr(S_t = j|I_{t-1}; \boldsymbol{\Theta}) \times c_j(u_{1,t}, u_{2,t}; \boldsymbol{\theta}_j^C, \boldsymbol{\Pi}|I_{t-1})]]. \tag{12}$$

To execute Markov-switching dependence, we apply a Hamilton filter to the copula segment. For an excellent overview of this procedure, please refer to Hamilton (1990, 1994).

The optimal inference and forecast for each period $t$ in the sample period can be found by iterating on the following pair of equations:

$$\hat{\boldsymbol{\xi}}_{t|t} = \left[\hat{\boldsymbol{\xi}}'_{t|t-1}\boldsymbol{\eta}_t\right]^{-1}\hat{\boldsymbol{\xi}}_{t|t-1} \odot \boldsymbol{\eta}_t, \tag{13}$$

$$\hat{\boldsymbol{\xi}}_{t+1|t} = \boldsymbol{\Pi}' \cdot \hat{\boldsymbol{\xi}}_{t|t}, \tag{14}$$

where $\odot$ is the Hadamard product, and $\boldsymbol{\eta}_t$ represents the density of the conditional copula in (12) given the state.[6] Precisely, $\boldsymbol{\eta}_t$ takes the form

$$\boldsymbol{\eta}_t = \begin{bmatrix} c_1(u_{1,t}(\boldsymbol{\theta}_1^M), u_{2,t}(\boldsymbol{\theta}_2^M); \boldsymbol{\theta}_1^C, \boldsymbol{\Pi}|I_{t-1}) \\ c_0(u_{1,t}(\boldsymbol{\theta}_1^M), u_{2,t}(\boldsymbol{\theta}_2^M); \boldsymbol{\theta}_0^C, \boldsymbol{\Pi}|I_{t-1}) \end{bmatrix}, \tag{15}$$

and we need standard uniformly distributed inputs for the copula density as indicated by the Canonical Maximum Likelihood (CML) approach.

The vector $\hat{\boldsymbol{\xi}}_{t|t}$ contains the probabilities of being in either state 1 or 0, given all the information up to the current period ($I_t$) and the parameter set $\boldsymbol{\theta}^C$. Analogously, $\hat{\boldsymbol{\xi}}_{t+1|t}$ holds the probabilities of being in either state at time $t+1$. Specifically, $\hat{\boldsymbol{\xi}}_{t|t}$ and $\hat{\boldsymbol{\xi}}_{t+1|t}$ take the forms

$$\hat{\boldsymbol{\xi}}_{t|t} = \begin{bmatrix} \Pr(S_t = 1|I_t; \boldsymbol{\theta}^M, \boldsymbol{\theta}_1^C, \boldsymbol{\Pi}) \\ \Pr(S_t = 0|I_t; \boldsymbol{\theta}^M, \boldsymbol{\theta}_0^C, \boldsymbol{\Pi}) \end{bmatrix}, \tag{16}$$

$$\hat{\boldsymbol{\xi}}_{t+1|t} = \begin{bmatrix} \Pr(S_{t+1} = 1|I_t; \boldsymbol{\theta}^M, \boldsymbol{\theta}_1^C, \boldsymbol{\Pi}) \\ \Pr(S_{t+1} = 0|I_t; \boldsymbol{\theta}^M, \boldsymbol{\theta}_0^C, \boldsymbol{\Pi}) \end{bmatrix}. \tag{17}$$

Using numerical methods, we can obtain the maximum likelihood estimates of the joint model

[6] Even if we know $\boldsymbol{\Theta}$, we cannot know which regime the process was in at each point in time, and the best we can do is to form a probabilistic inference.

parameters through either the Newton–Raphson, quasi-Newton or simplex methods. Since the marginal distributions are separable from the copula model, we use a two-step procedure for the estimation, namely the inference functions for margins (IFM) approach, see Joe and Xu (1996) and Joe (2014). In the first step, we estimate the marginal models and the ECDF of the standardized residuals from the marginal model. In the second step, we estimate the parameters of the mixture copulas ($\boldsymbol{\theta}_1^C, \boldsymbol{\theta}_0^C$) and the transition matrix ($\mathbf{\Pi}$) with inputs of the ECDFs estimated from the first step. Mathematically, the two-step estimation can be expressed as follows:

$$\hat{\boldsymbol{\theta}}^M = \underset{\boldsymbol{\theta}^M \in \mathbf{\Theta}^M}{\arg\max} \sum_{i=1}^{2} L_i(\boldsymbol{\theta}_i^M; I_{i,T}), \tag{18}$$

$$\hat{\boldsymbol{\psi}} = \underset{\boldsymbol{\psi} \in \mathbf{\Psi}}{\arg\max}\, L_C(\hat{u}_{1,t}, \hat{u}_{2,t}, \boldsymbol{\theta}^C, \mathbf{\Pi}; I_T), \tag{19}$$

where $\boldsymbol{\psi} = (\boldsymbol{\theta}^C, \mathbf{\Pi})$. $\mathbf{\Theta}^M$ and $\mathbf{\Psi}$ denote the sets of possible values of $\boldsymbol{\theta}^M$ and $\boldsymbol{\psi}$, respectively. As shown by Joe (1997), under certain regularity conditions, the IFM estimator exists and is consistent and asymptotically normal. For a discussion on efficiency, refer to Joe (2005) and Patton (2009).

In general, we have

$$\sqrt{T}(\hat{\mathbf{\Theta}} - \mathbf{\Theta}_0) \xrightarrow{d} N(\mathbf{0}, -\mathbf{H}^{-1}(\mathbf{\Theta}_0)), \text{ for } t \to \infty,$$

where $\hat{\mathbf{\Theta}}$ is any consistent estimator of the full parameter set, and $\mathbf{H}^{-1}(\mathbf{\Theta}_0)$ is the inverse of the Hessian (Bierens, 2004; Cherubini et al., 2004; Martin et al., 2013). For a derivation of the above result, refer to Joe (2014). We employ the delta method to compute the standard errors of the joint parameter estimates. Assume that $\{\hat{\boldsymbol{\Theta}}_N\}$ is a sequence of $V \times 1$ random vectors such that

$$\sqrt{T}(\hat{\boldsymbol{\Theta}}_N - \boldsymbol{\Theta}_0) \xrightarrow{d} N(\mathbf{0}, -\mathbf{H}^{-1}(\boldsymbol{\Theta}_0)), \text{ for } t \to \infty.$$

Let $\mathbf{g} : \mathbb{R}^V \to \mathbb{R}^W$ be a continuously differentiable function in $W$ dimensions with respect to $\boldsymbol{\Theta}$, then

$$\sqrt{T}(\mathbf{g}(\hat{\boldsymbol{\Theta}}_N) - \mathbf{g}(\boldsymbol{\Theta}_0)) \xrightarrow{d} N(\mathbf{0}, [\mathbf{J}_{\mathbf{g}}(\boldsymbol{\Theta}_0)][-\mathbf{H}^{-1}(\boldsymbol{\Theta}_0)][\mathbf{J}_{\mathbf{g}}(\boldsymbol{\Theta}_0)]'), \text{ for } t \to \infty,$$

where $\mathbf{J}_{\mathbf{g}}$ is the Jacobian of $\mathbf{g}$, i.e., the $W \times V$ matrix of partial derivatives of $\mathbf{g}$ in relation to the entries of $\boldsymbol{\Theta}$. For a more detailed account, refer to Wooldridge (2010), Yee (2015), and Hansen (2022).

# 3 Data

We consider five potential safe haven currencies, which involve four exchange rates in terms of one USD, namely the EUR, the GBP, the JPY, and the CHF per unit of the USD. Thus, the base currency is the USD, and the others are the quote currencies. The data are obtained from Thomson Reuters. In addition, we gather data on the Federal Reserve dollar indexes from the Federal Reserve System website. The broad dollar index (Broad), is constructed using the currencies of the most important US trading partners by volume of bilateral trade.[7] We chose the broad index for our principal analysis since it is more inclusive.

The market risk proxies are the VIX, the Euro Stoxx 50 Volatility Index (V2X), and the Volatility

[7]Two sub-indexes split the broad index into advanced foreign economies (AFE) and emerging market economies (EME).

Index on the Swiss Market Index (VSMI), sourced from CBOE, SIX Group, and Qontigo, respectively. The VIX is constructed from the implied volatility of option prices on the S&P 500 over the next 30 days. The V2X and the VSMI are created similarly to the VIX but based on 50 blue chip euro zone stocks and the 20 largest Swiss stocks, respectively. The frequency of the data is daily, and the time span is from January 01, 1999, to June 04, 2024.

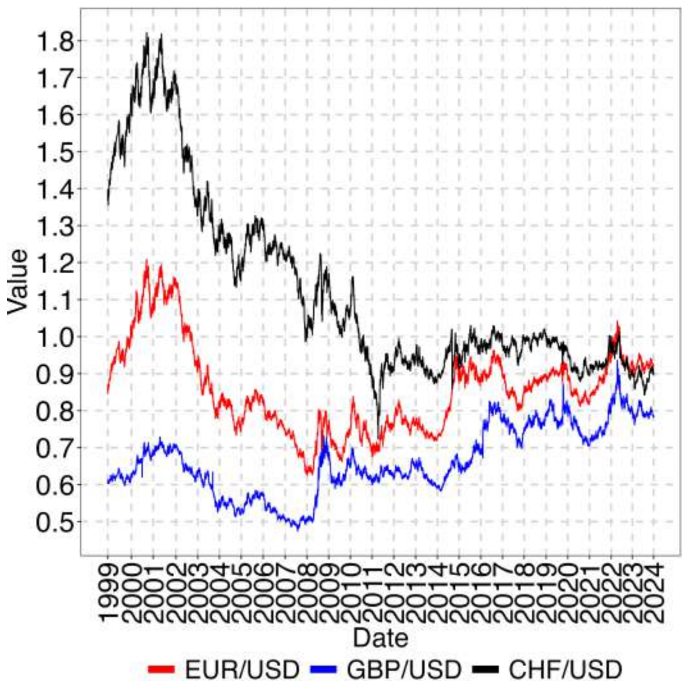
Figure 2a plots the per USD exchange rates for the euro, the pound, and the CHF, whereas Figure 2b displays the exchange rate of yen per USD. The graphs show that all four exchange rates fluctuated substantially. The euro, the pound, and the franc depreciated against the USD around the dot-com bubble, the financial crisis of 2007–08, and the onset of the pandemic, giving evidence contrary to safe havens or of less potent safe havens than the USD. On the other hand, the Japanese yen appreciated against the USD for the periods of 1999–2000, 2007–2011, and 2019–2020, showing evidence that it is a more vital safe haven than the USD during these periods. Figure 2c provides the three USD indexes, which are highly correlated. We can also see that the USD indexes appreciated during several crises or market turmoil periods, including the 2001 tech bubble, the 2007–2008 financial crisis, and the pandemic period, which is evidence of a safe haven currency.

Figure 3 plots the volatility indexes. It is clear from the graph that the three volatility indexes are highly correlated. Table 1 further details the correlations between these indexes. As depicted in Figure 3 and consistent with existing literature (see Habib and Stracca (2012), Fatum and Yamamoto (2016), and Wong and Fong (2018)), the correlations between VIX and the other two market risk proxies are very high, almost reaching 1. Therefore, we use VIX as the proxy for market risk.

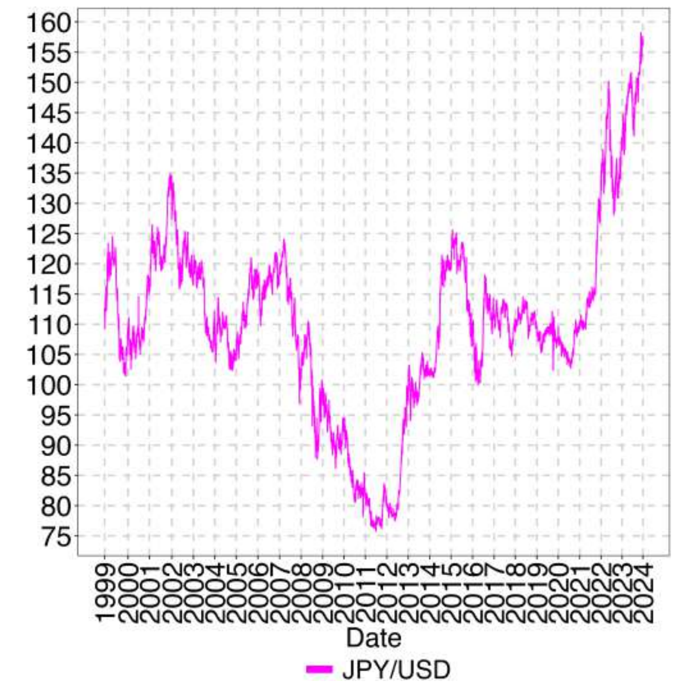
Table 2 presents the summary statistics of the variables. The exchange rates and the trade-weighted USD (TW-USD) are their returns computed by taking log differences and are expressed in

Figure 2: Data graph

(a) Euro, pound, franc




(b) Yen




(c) Trade-weighted USD

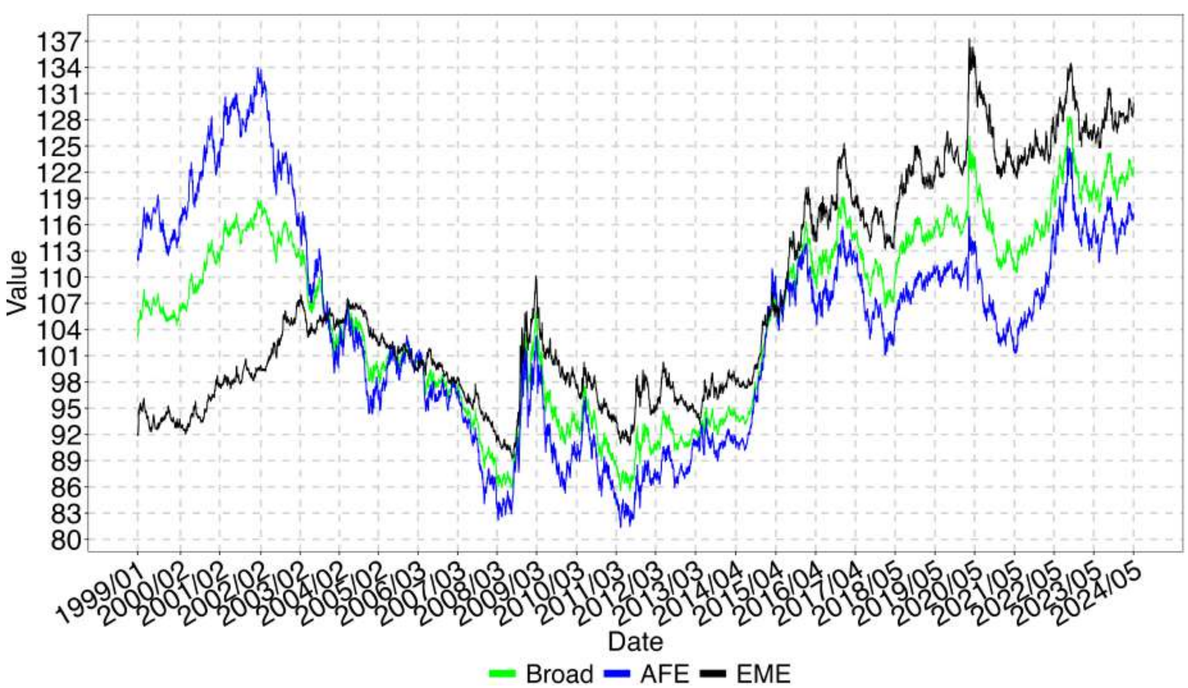


Figure 3: Volatility indexes

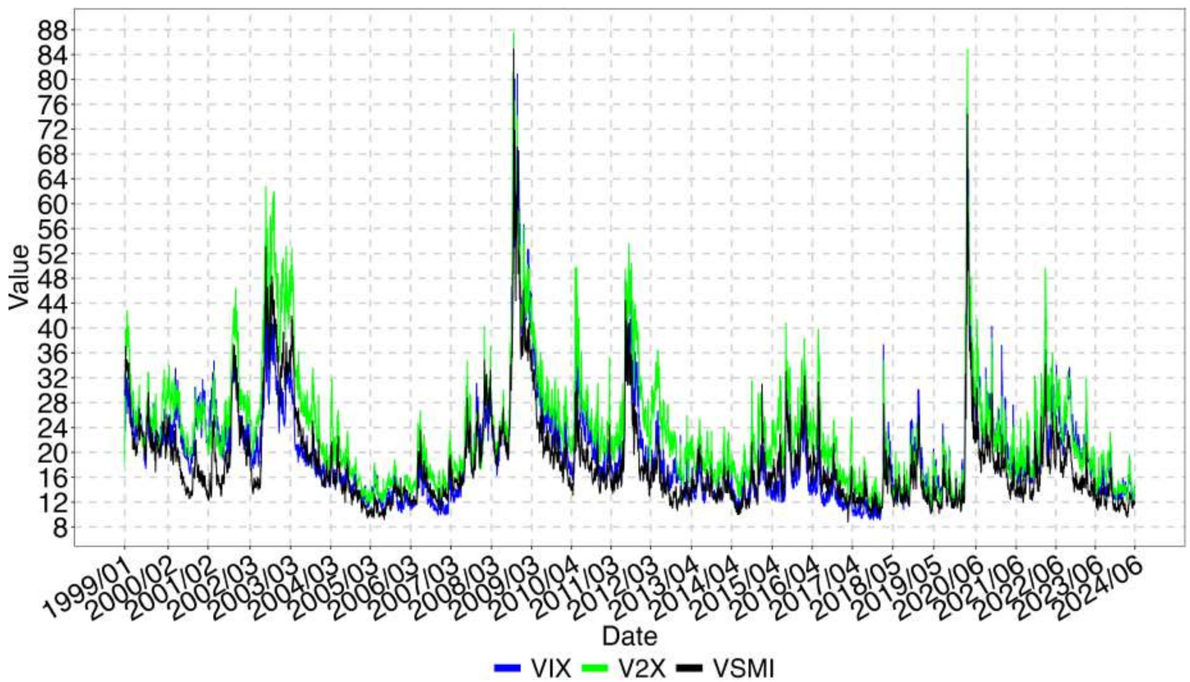



Table 1: Correlations between volatility indexes

| | VIX | V2X | VSMI |
|---|---|---|---|
| VIX | 1.000 | 0.900 | 0.894 |
| V2X | 0.900 | 1.000 | 0.948 |
| VSMI | 0.894 | 0.948 | 1.000 |

*Notes*: The VIX is the CBOE's volatility index. V2X and VSMI denote the volatility indexes of the Euro Stoxx 50 and Swiss Market Index, respectively.

Table 2: Summary statistics of the variables

| | Mean | Std | Skewness | Kurtosis | Min | Max | JB |
|---|---|---|---|---|---|---|---|
| VIX | 20.084 | 8.405 | 2.147 | 8.001 | 9.140 | 82.690 | $21,618.500$ |
| ln(VIX) | 2.930 | 0.360 | 0.596 | 0.348 | 2.213 | 4.415 | 403.877 |
| EUR/USD | 0.001 | 0.594 | -0.034 | 2.582 | -4.617 | 3.844 | $1,750.774$ |
| GBP/USD | 0.004 | 0.626 | 0.625 | 22.962 | -7.943 | 8.410 | $138,647.300$ |
| CHF/USD | -0.007 | 0.667 | -2.507 | 76.454 | -17.137 | 8.929 | $1,539,036.000$ |
| JPY/USD | 0.005 | 0.642 | -0.356 | 5.985 | -5.562 | 5.854 | $9,526.753$ |
| TW-USD | 0.003 | 0.327 | 0.016 | 4.161 | -2.553 | 1.893 | $4,541.310$ |

*Notes*: The daily data is from January 01, 1999, to June 04, 2024. We denote the Jarque–Bera statistic with JB, and the rejection of the null that the data is distributed normally at a 1% significance level is signified by three asterisks.

percentages. Following the literature, we scale the VIX by taking its log value, ln(VIX), which is used in the empirical analysis hereafter. All variables exhibit positive excess kurtosis, with the CHF/USD and the GBP/USD showing extremely high excess kurtosis of 76.45 and 22.96, respectively.[8] This indicates that all variables are highly leptokurtic and have fat tails. Also, all currency returns are negatively skewed. According to Coudert et al. (2014) and Hossfeld and MacDonald (2015), the market is considered to be in turmoil if the VIX is above 30. In our sample period, the VIX ranges from 9.14 to 82.69, suggesting periods of high market turmoil. The skewness and kurtosis of the variables suggest that our variables are not normally distributed, a conclusion supported by the Jarque–Bera normality test results presented in the last column of the table.

Table 3 provides three dependence measures between exchange rate returns and the VIX. All dependence measures are positive except for the yen per USD and the first entry of column 3, indicating that when the VIX increases, the USD tends to appreciate against the euro, the pound, or the franc, while the yen appreciates against the USD. This suggests that the JPY may be a

[8]The CHF/USD is very negatively skewed (-2.548).

Table 3: Dependence measures between the currencies and ln(VIX)

| | EUR/USD | GBP/USD | CHF/USD | JPY/USD | TW-USD |
|---|---|---|---|---|---|
| Linear correlation | 0.022 | 0.049 | -0.002 | -0.047 | 0.060 |
| Spearman's $\rho$ | 0.013 | 0.027 | 0.002 | -0.041 | 0.037 |
| Kendall's $\tau$ | 0.008 | 0.018 | 0.001 | -0.028 | 0.025 |

*Notes*: The dependence measures provided above are the nonparametric versions. For every column, one input is always ln(VIX), and the second input varies based on the variable given in the header.

stronger safe haven currency compared to the USD.

# 4 Empirical results

Table 4 provides the results for the marginal models. The parameter estimates of the GARCH and GED terms are significant for all variables (the AR terms are not statistically significant for all variables).[9][10] Thus, the marginal models remove the variables' serial correlations, heteroskedasticity and fat tails. This well prepares the standardized residuals from the marginal models for the joint copula model.

Figure 4 presents the joint empirical cumulative distribution function (ECDF) of exchange rate returns and the VIX.[11] In this figure, the concentration of observations in each corner of the bivariate

[9]The characterization of left skewness ($\zeta < 1$) seems to be warranted in the cases of the CHF/USD and the JPY/USD. This implies that large appreciations of these currencies occur more frequently than the converse, which is compatible with Hossfeld and MacDonald's (2015) observations. The statistically significant value of 1.293 ($\zeta$) under the VIX specification indicates right skewness, the highest among all formulations.

[10]The estimates of the shape parameter ($\nu > 0$) suggest that our dataset has less weight in the tail than the double-exponential distribution.

[11]Assume we have $T$ observations $\mathbf{x}_t = (x_{1,t}, ...x_{d,t})$ where $t \in \{1, ..., T\}$ and $\ell \in \{1, ..., d\}$. Then, $\hat{F}_\ell$ is estimated by (9). The estimated margins are used to create a sample, i.e., $\mathbf{U}_t = (\hat{F}_1(x_{1,t}), ..., \hat{F}_d(x_{d,t})) = \frac{1}{T+1}(r_{1,t}, ..., r_{d,t})$. In the above equation, $r_{\ell,t}$ denotes the rank of $x_{\ell,t}$ among all $\mathbf{x}'_\ell = (x_{\ell,1}, ..., x_{\ell,T})'$, and the division of $T + 1$ occurs to ensure that the sample lies in the interior $(0, 1)^d$ of the unit hypercube. The purpose of nonparametrically estimating the margins is to give us an initial idea of the dependence structure before we explore the parametric joint model.

Table 4: AR(1)-GARCH(1,1) with a skewed GED innovation

| | *Response variable:* | | | | | |
|---|---|---|---|---|---|---|
| | ln(VIX) | EUR/USD | GBP/USD | CHF/USD | JPY/USD | TW-USD |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\mu$ | 2.950*** | −0.001 | 0.001 | −0.007*** | 0.008 | 0.001 |
| | (0.060) | (0.007) | (0.009) | (0.001) | (0.006) | (0.004) |
| $\phi$ | 0.983*** | −0.015 | −0.002 | −0.010*** | −0.030*** | 0.027** |
| | (0.002) | (0.013) | (0.018) | (0.002) | (0.007) | (0.012) |
| $\omega$ | 0.0004*** | 0.001*** | 0.005 | 0.0004*** | 0.004 | 0.001*** |
| | (0.0001) | (0.0004) | (0.016) | (0.001) | (0.004) | (0.0002) |
| $\alpha$ | 0.142*** | 0.034*** | 0.047 | 0.050*** | 0.050* | 0.043*** |
| | (0.015) | (0.002) | (0.090) | (0.003) | (0.028) | (0.005) |
| $\beta$ | 0.771*** | 0.964*** | 0.938*** | 0.900*** | 0.941*** | 0.951*** |
| | (0.025) | (0.0002) | (0.134) | (0.0002) | (0.035) | (0.004) |
| $\zeta$ | 1.272*** | 1.017*** | 1.033*** | 0.930*** | 0.969*** | 1.040*** |
| | (0.025) | (0.016) | (0.029) | (0.0002) | (0.015) | (0.020) |
| $\nu$ | 1.293*** | 1.512*** | 1.285*** | 2.000*** | 1.218*** | 1.452*** |
| | (0.041) | (0.042) | (0.082) | (0.0003) | (0.042) | (0.046) |
| $T$ | 6,288 | 6,288 | 6,288 | 6,288 | 6,288 | 6,288 |
| LL | 8,606.711 | −5,110.432 | −5,118.736 | −7,968.419 | −5,459.429 | −1,213.707 |
| AIC | −2.735 | 1.628 | 1.630 | 2.537 | 1.739 | 0.388 |
| BIC | −2.728 | 1.635 | 1.638 | 2.544 | 1.746 | 0.396 |

*Notes*: *, **, and *** denote significance at 10%, 5%, and 1%, respectively. In sequential order, the first set of parameters is the mean intercept ($\mu$) and the AR ($\phi$) coefficient of the AR(1) model. The next set of parameters is for the GARCH model: the dispersion intercept ($\omega$), the ARCH term ($\alpha$), and the GARCH term ($\beta$). The skewness and the shape parameters are denoted by $\zeta$ and $\nu$, respectively. The standard errors are in parentheses. $T$ is the number of observations. LL, AIC, and BIC denote the estimated log-likelihood value, the Akaike information criterion, and the Bayesian information criterion, respectively.

distribution indicates the density of the tails. The cutoff for the left (lower) side of the distribution is the 10%-quantile, and for the right (upper) side is the 90%-quantile. For instance, the left–upper corner corresponds to the tail region of [0%–10%-quantiles, and 90%–100%-quantiles].[12] Figure 4 clearly shows that the right–upper and the right–lower regions are denser than the other two sections. Since our exchange rates are in terms of USD, the upper right corner is associated with USD appreciation when the VIX is high, while the lower right corner signifies non-USD currency appreciation during high VIX periods. Therefore, the right–upper or right–lower corners support the safe haven properties of the base and quote currencies, respectively.

The graphs reveal that the percentage of observations falling into the right–upper corner is higher than those in the right–lower corner for all pairs except for JPY/USD. For example, the values associated with the right–upper and right–lower corners for EUR/USD are 0.0189 and 0.0159, respectively. The comparison of mass in the right–upper and right–lower corners suggests that during periods of extremely high market risk, the USD appreciates more than the euro, the pound, and the CHF, while the JPY appreciates more than the USD, indicating the stronger safe haven status of the USD compared to the euro, the pound, and the franc, with the JPY being an even stronger safe haven than the USD.[13][14][15] These results are consistent with Fratzscher (2009), De Bock and de Carvalho Filho (2015), Fatum and Yamamoto (2016), and Tachibana (2018).[16] Although the

[12]The most sparsely populated region is the left–upper corner, which is related to the scenario when there is low perceived risk and simultaneous dollar appreciation. For the traded-weighted USD (Figure 4e), the left–upper region has a value of 0.54% associated with it, which is lower than 0.97% of the left–lower region.

[13]When the value of the VIX is high, it usually means market underperformance and financial stress (Hakkio et al., 2009; Bekaert et al., 2014).

[14]When global risk aversion is heightened, the trade-weighted USD appreciates more than the basket of currencies used to construct the broad index, which includes the JPY, consistent with figures 4a–4d.

[15]The strength of the USD can be explained by nation-states' high financial exposure to the US, dollar shortages, and the reversal of carry trades (Fratzscher, 2009; McCauley and McGuire, 2009).

[16]The funding stage of currency carry trade can partially help explain why we observe a cluster in the lower left corner since, during booms, long positions are taken in high-yield currencies by shorting low-yield ones (Brunnermeier et al., 2008; Habib and Stracca, 2012; Menkhoff et al., 2012; Coudert et al., 2014).

USD significantly overshadows the safe haven behaviour of the euro, the pound, and, to a lesser extent, the franc, these currencies also exhibit their own safe haven characteristics, as evidenced by the clustering of observations in the right–lower corner.

Table 5 provides the estimates of our joint model.[17] Clearly, a vast majority of the parameter estimates are statistically significant. The two most relevant parameters used to measure the safe haven properties are $\lambda_2^{RR}$ and $\lambda_3^{RL}$, which are significant for all currencies. $\lambda_2^{RR}$ and $\lambda_3^{RL}$ represent when the market risk is exceptionally high, the base currency (the USD) appreciates, or the quote currency appreciates, respectively. The significance of both parameters indicates that all currencies show safe haven properties. In addition, except for the JPY, the estimates of the tail dependence coefficient $\lambda_2^{RR}$ are larger than $\lambda_3^{RL}$, where the former is associated with the safe haven behaviour of the USD and the latter with that of the quote currencies. For example, the estimates of $\lambda_2^{RR}$ are 0.051 and 0.097 for the euro and the pound, respectively, much stronger than $\lambda_3^{RL}$, which are 0.038 and 0.044, respectively. This indicates that the USD is a safe haven currency relative to the euro and the pound. Our joint parameter estimates for the trade-weighted USD are consistent with the safe haven nature of the dollar since they indicate that the value of the USD index increases when there is heightened global risk aversion, as suggested by the parameter estimate of $\lambda_2^{RR}$ (0.090) relative to all other tail dependence coefficients.[18][19]

For the CHF, the estimates of $\lambda_2^{RR}$ and $\lambda_3^{RL}$ are 0.029 and 0.019, respectively, which do not

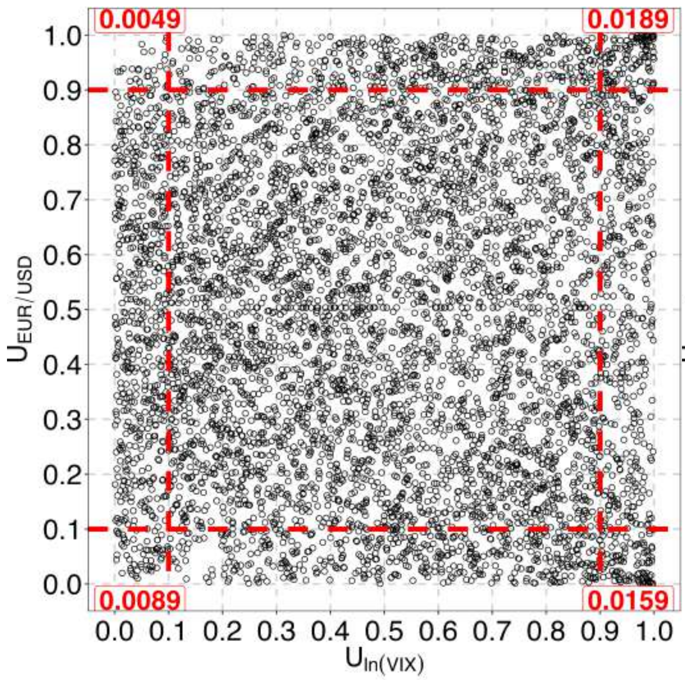
[17] The parameter estimate of $\lambda_4^{LR}$ is statistically insignificant for all pairs, which is consistent with our previous analysis that we do not typically observe an appreciation of the USD when the VIX is low.

[18] The funding stage of currency carry trade can partially help explain why we observe a cluster in the lower left corner since, during booms, long positions are taken in high-yield currencies by shorting low-yield ones (Brunnermeier et al., 2008; Habib and Stracca, 2012; Menkhoff et al., 2012; Coudert et al., 2014). When global risk aversion is heightened, the trade-weighted USD appreciates more than the basket of currencies used to construct the broad index, which includes the JPY, consistent with figures 4a–4d.

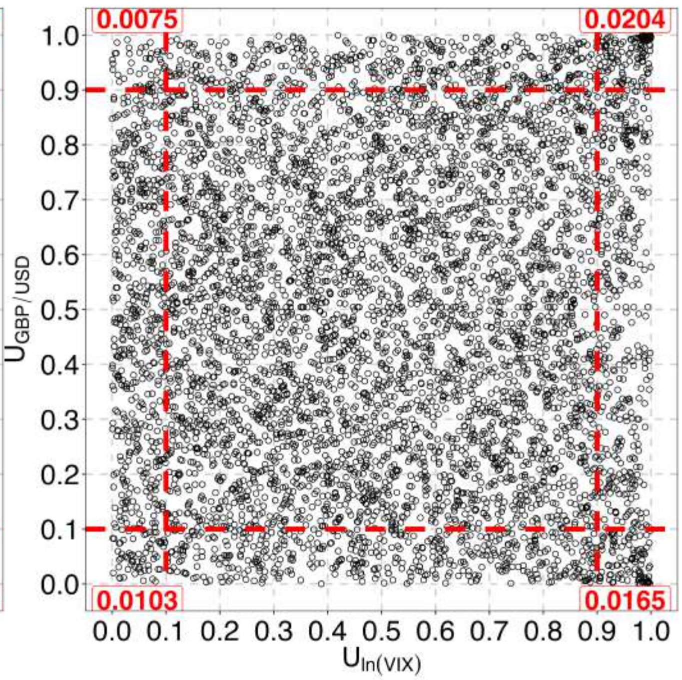
[19] Our results are compatible with the USD being a funding currency since it can depreciate when markets are relatively tranquil, as indicated by the statistically significant estimates of $\lambda_1^{LL}$.

Figure 4: ECDF

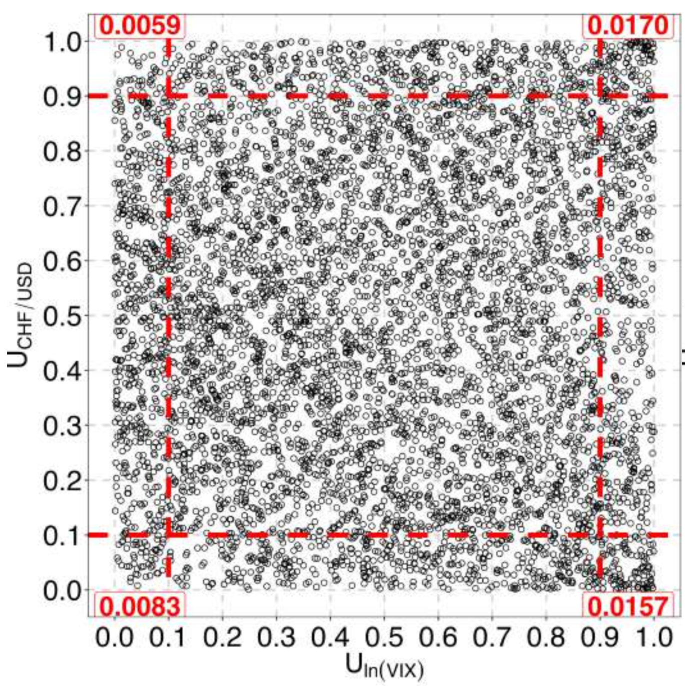
(a) EUR/USD




(b) GBP/USD




(c) CHF/USD




(d) JPY/USD

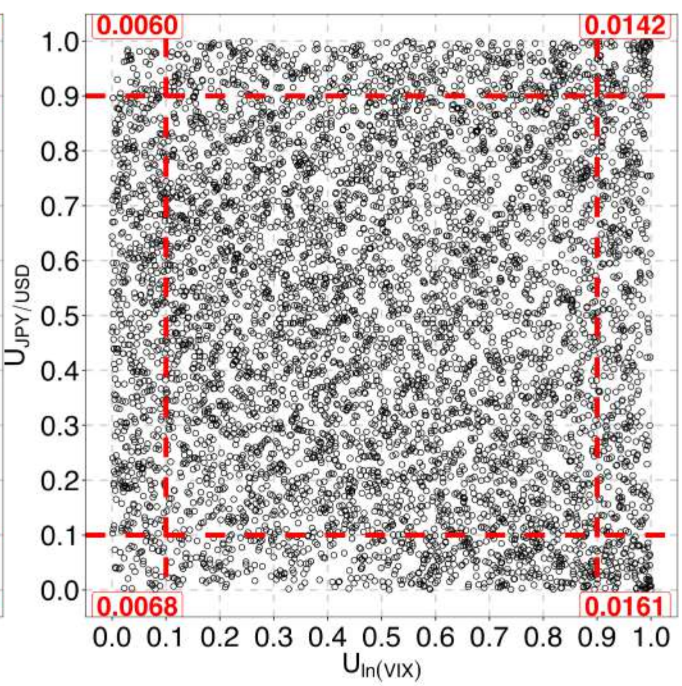
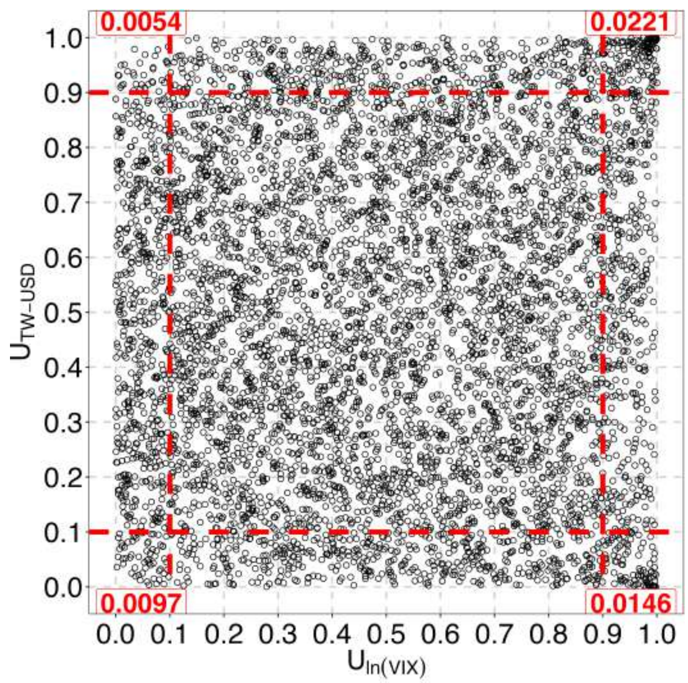


Figure 4: ECDF (*continued*)

(e) TW-USD



substantially differ, implying that it is slightly weaker than the USD as a safe haven (also, please refer to Figure 4c for the joint ECDF estimates). On the other hand, the JPY is the anomaly whose $\hat{\lambda}_2^{RR}$ (0.028) is significantly below $\hat{\lambda}_3^{RL}$ (0.046), indicating that the JPY displays considerable safe haven attributes. In fact, the JPY is the only currency that is not overpowered by the safe haven character of the USD. These results are consistent with our prior joint ECDF results.

These findings are consistent with previous studies, such as Coudert et al. (2014), Fatum and Yamamoto (2016), and Wong and Fong (2018), regarding the status or strength of the JPY as a safe haven currency.[20] However, there is disagreement regarding the CHF's classification as a safe haven, although there is consensus that the USD possesses safe haven properties. Our results suggest that the CHF exhibits weaker safe haven behaviour than the yen and the USD.[21] The significance of $\lambda_3^{RL}$ for the euro and the pound indicates that they are both safe havens, though they are weaker than the USD and the JPY. This partially differs from Ranaldo and Söderlind's (2010) findings, which suggest that the euro has weaker safe haven characteristics than the CHF and the JPY, and the pound may not be considered as a safe haven.

The correlation coefficient ($\rho_k$) reflects the linear dependence observed in various quadrants. The lower estimated value for $\rho_2$ than $\rho_3$ in column 5 of Table 5 suggests that, on average, the appreciation of the USD during periods of increasing volatility is generally weaker compared to the quote currencies. However, the stronger tail dependence coefficient, $\lambda_2^{RR}$, relative to $\lambda_3^{RL}$, indicates that the extreme appreciation of the USD when market risk is exceptionally high is much stronger than that of the quote currencies, suggesting a stronger safe haven status of the USD. Furthermore,

[20]Because of the liquidation of carry trade positions in times of crises, Hossfeld and MacDonald (2015) put forward that the yen should be considered a carry funding currency rather than a safe haven.

[21]De Bock and de Carvalho Filho (2015) observe that the USD depreciates against the CHF and the JPY during risk-off episodes when risky positions are liquidated or unwound.

Table 5: Joint model estimates

| | Volatility and currency pairs: | | | | |
|---|---|---|---|---|---|
| | EUR/USD | GBP/USD | CHF/USD | JPY/USD | TW-USD |
| | (1) | (2) | (3) | (4) | (5) |
| $\rho_1$ | 0.160*** | 0.156*** | 0.161*** | 0.098*** | 0.157*** |
| | (0.006) | (0.013) | (0.006) | (0.027) | (0.009) |
| $\lambda_1^{LL}$ | 0.017*** | 0.017*** | 0.015*** | 0.003 | 0.015*** |
| | (0.002) | (0.005) | (0.002) | (0.005) | (0.003) |
| $\rho_2$ | 0.187*** | 0.245*** | 0.230*** | 0.182*** | 0.231*** |
| | (0.023) | (0.029) | (0.085) | (0.065) | (0.024) |
| $\lambda_2^{RR}$ | 0.051*** | 0.097*** | 0.029** | 0.028* | 0.090*** |
| | (0.018) | (0.025) | (0.014) | (0.016) | (0.022) |
| $w_1$ | 0.356*** | 0.382*** | 0.310*** | 0.606*** | 0.343*** |
| | (0.040) | (0.047) | (0.040) | (0.135) | (0.041) |
| $\Pi_{11}$ | 0.886*** | 0.887*** | 0.882*** | 0.887*** | 0.887*** |
| | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| $\rho_3$ | 0.280*** | 0.410*** | 0.138*** | 0.242*** | 0.335*** |
| | (0.080) | (0.104) | (0.020) | (0.060) | (0.084) |
| $\lambda_3^{RL}$ | 0.038* | 0.044*** | 0.019* | 0.046* | 0.031*** |
| | (0.020) | (0.011) | (0.011) | (0.026) | (0.011) |
| $\rho_4$ | 0.093*** | 0.105*** | 0.071*** | 0.067*** | 0.106*** |
| | (0.015) | (0.017) | (0.024) | (0.025) | (0.014) |
| $\lambda_4^{LR}$ | 0.003 | 0.007 | 0.000 | 0.000 | 0.003 |
| | (0.002) | (0.008) | (0.001) | (0.001) | (0.002) |
| $w_2$ | 0.184*** | 0.113*** | 0.216** | 0.300*** | 0.108*** |
| | (0.061) | (0.032) | (0.092) | (0.087) | (0.033) |
| $\Pi_{00}$ | 0.962*** | 0.962*** | 0.962*** | 0.962*** | 0.962*** |
| | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| LL | -10,053.21 | 7,957.468 | -10,152.73 | -10,075.38 | 7,859.611 |

*Notes*: *, **, and *** denote significance at 10%, 5%, and 1%, respectively. $\rho_k$ represents the correlation coefficient. The parameter $\lambda_k^{\cdot\cdot}$ denotes the tail dependence coefficient, where the superscript $L$ and $R$ signify the left and right tails, respectively. The weights of the mixture copulas are denoted by $w_1$ and $w_2$. $\Pi_{11}$ and $\Pi_{00}$ are the two transition probabilities between two consecutive positive dependence regimes and two consecutive negative dependence regimes, respectively. The standard errors are in round brackets. LL denotes the estimated log-likelihood value.

the transition probability of remaining in the positive dependence state is higher than that in the negative dependence regime, indicating more extended periods of USD appreciation compared to the quote currencies when the VIX is high. This is also supported by the higher weight ($w_1$) in the right–upper dependence quadrant compared to the weight ($w_2$) in the right–lower dependence quadrant. Therefore, our method of using tail dependence and dependence-switching copula models to identify safe havens is more direct and practical.

Figures 5a–5e present the smoothing correlations computed following Wang et al. (2013, 2018) (also see Kim (1994) and Kim et al. (1999)).[22] Clearly shown in the graphs, the correlation is positive during significant market events such as the 2000–2002 high-tech bubble, the financial crisis of 2007–08, and the onset of the pandemic in 2020. This again suggests that the USD demonstrates safe haven characteristics during these major market turmoil and crisis periods.

# 5 Conclusion

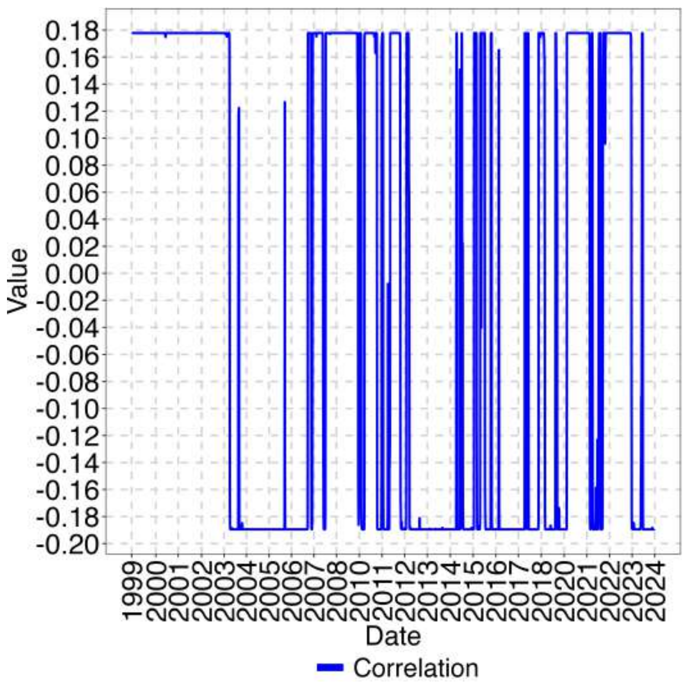
In this paper, we examined the extent to which five major currencies: the US dollar, USD, the Japanese yen, JPY, the Swiss franc, CHF, the Euro, EUR, and the British pound, GBP, can serve as safe haven currencies in the face of economic shocks with global effects. The empirical analysis was carried out in the context of a flexible dependence switching copula, DSC, model between each of the four exchange rates relative to the US dollar and the VIX index as a proxy of global volatility or global market risk.

The DSC model is more flexible and offers new empirical insights into the properties of safe

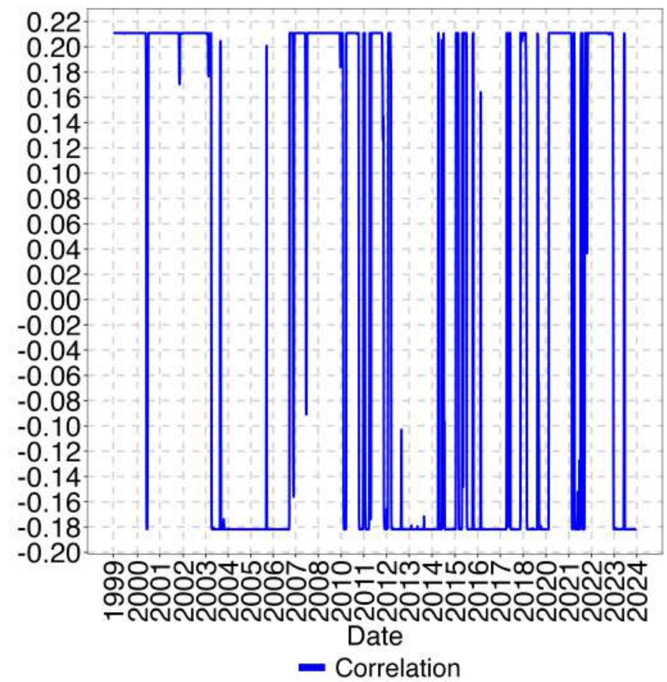
[22]For the positive dependence regime we have the following Kendall's $\tau$, i.e., $\tau^1 = w_1\left[\frac{\chi_1}{2+\chi_1}\right] + (1-w_1)\left[\frac{\chi_2}{2+\chi_2}\right]$. Similarly, for the negative dependence regime we have $\tau^0 = w_2\left[\frac{\chi_3}{2+\chi_3}\right] + (1-w_2)\left[\frac{\chi_4}{2+\chi_4}\right]$. The smoothing correlation is given by $\rho_{sm} = p_{1,sm}\sin\left(\frac{\pi\times\tau^1}{2}\right) - p_{0,sm}\sin\left(\frac{\pi\times\tau^0}{2}\right)$, where $p_{\cdot,sm}$ is the smoothed probability.

Figure 5: Smoothing correlation

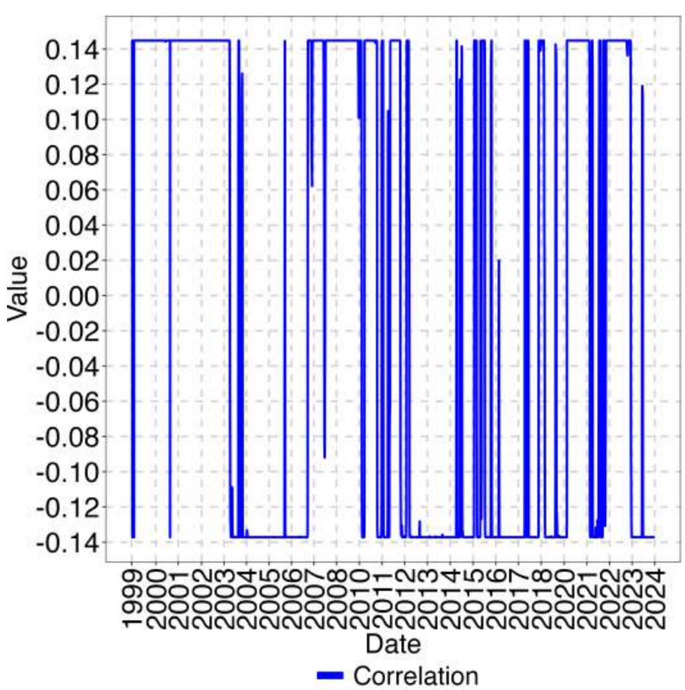
(a) EUR/USD




(b) GBP/USD




(c) CHF/USD




(d) JPY/USD

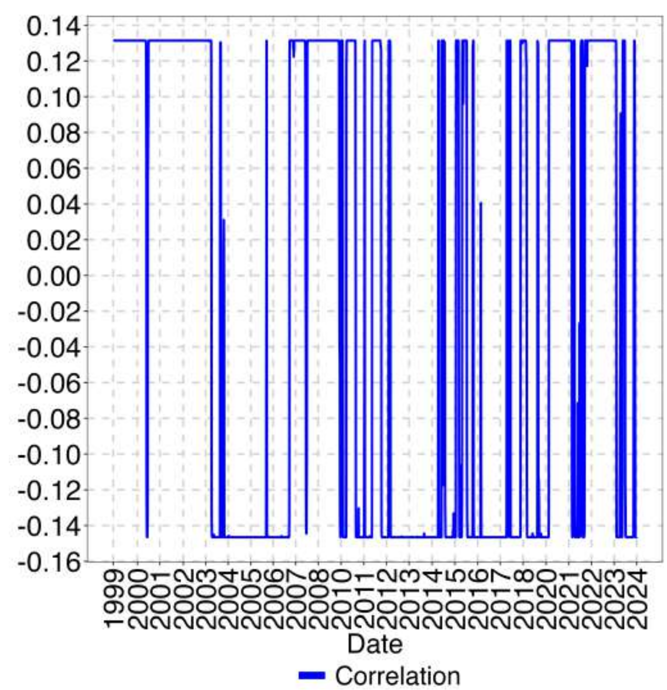
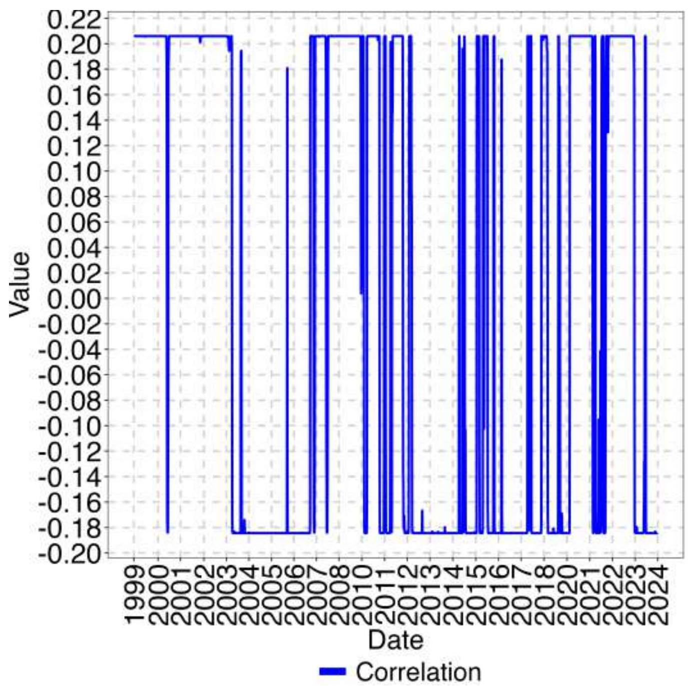


Figure 5: Smoothing correlation (*continued*)

(e) TW-USD



haven currencies relative to the regression and copula models used in the literature. Unlike linear regression models, the DSC approach allows us to capture currency value changes under extreme market conditions and economic downturns, which is directly tied to the concept of a safe haven currency. Further, unlike other copula models, the DSC model does not impose any directional restrictions on the joint dependence between exchange rates and the VIX. Thus, it can capture the relative strength of the safe haven property of a currency.

The empirical results indicate that the USD, the JPY and the CHF appear to be safe haven currencies. In terms of their relative strength, the USD is a strong, safe haven currency relative to the other currencies, except the JPY, which appears to be the strongest safe haven currency. On the other hand, the EUR and the GBP are the weakest safe haven currencies. These insights can be helpful to individual investors and policymakers in their efforts to mitigate the risk and avoid the detrimental effects of unexpected currency fluctuations.

For future work, the DSC model can be applied in different contexts with more currencies than the five major ones studied in the present paper. It is also flexible enough to study not only the safe properties of a set of currencies but also their carry trade and hedging properties.

## References


Aas, K. (2004), 'Modelling the dependence structure of financial assets: A survey of four copulas', *Research Report SAMBA/22/04, Norwegian Computing Center.* .

Bekaert, G., Ehrmann, M., Fratzscher, M. and Mehl, A. (2014), 'The global crisis and equity market contagion', *The Journal of Finance* **69**(6), 2597–2649.

Bierens, H. J. (2004), *Introduction to the mathematical and statistical foundations of econometrics*, Cambridge University Press.

Bollerslev, T. and Wooldridge, J. M. (1992), 'Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances', *Econometric reviews* **11**(2), 143–172.

Brunnermeier, M. K., Nagel, S. and Pedersen, L. H. (2008), 'Carry trades and currency crashes', *NBER macroeconomics annual* **23**(1), 313–348.

Caillault, C. and Guegan, D. (2005), 'Empirical estimation of tail dependence using copulas: application to asian markets', *Quantitative Finance* **5**(5), 489–501.

Calvo, G. A. (2002), 'On dollarization', *Economics of transition* **10**(2), 393–403.

Carr, P. and Wu, L. (2006), 'A tale of two indices', *The Journal of Derivatives* **13**(3), 13–29.

Chen, X. and Fan, Y. (2006), 'Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification', *Journal of econometrics* **135**(1-2), 125–154.

Cherubini, U., Luciano, E. and Vecchiato, W. (2004), *Copula methods in finance*, John Wiley & Sons.

Collin-Dufresn, P., Goldstein, R. S. and Martin, J. S. (2001), 'The determinants of credit spread changes', *The Journal of Finance* **56**(6), 2177–2207.

Coudert, V., Guillaumin, C., Raymond, H. et al. (2014), Looking at the other side of carry trades: Are there any safe haven currencies?, Technical report.

De Bock, R. and de Carvalho Filho, I. (2015), 'The behavior of currencies during risk-off episodes', *Journal of International Money and Finance* **53**, 218–234.

De Santis, R. A. (2012), 'The euro area sovereign debt crisis: safe haven, credit rating agencies and the spread of the fever from greece, ireland and portugal', *No. 1419. European Central Bank* **NA**.

Embrechts, P., McNeil, A. and Straumann, D. (2002), 'Correlation and dependence in risk management: properties and pitfalls', *Risk management: value at risk and beyond* **1**, 176–223.

Fatum, R. and Yamamoto, Y. (2016), 'Intra-safe haven currency behavior during the global financial crisis', *Journal of International Money and Finance* **66**, 49–64.

Fernández, C. and Steel, M. F. (1998), 'On bayesian modeling of fat tails and skewness', *Journal of the american statistical association* **93**(441), 359–371.

Fratzscher, M. (2009), 'What explains global exchange rate movements during the financial crisis?', *Journal of International Money and Finance* **28**(8), 1390–1407.

Garcia, R. and Tsafack, G. (2011), 'Dependence structure and extreme comovements in international equity and bond markets', *Journal of Banking & Finance* **35**(8), 1954–1970.

Gonzalez-Perez, M. T. (2015), 'Model-free volatility indexes in the financial literature: A review', *International Review of Economics & Finance* **40**, 141–159.

Grisse, C. and Nitschka, T. (2015), 'On financial risk and the safe haven characteristics of swiss franc exchange rates', *Journal of Empirical Finance* **32**, 153–164.

Gyntelberg, J. and Schrimpf, A. (2011), 'Fx strategies in periods of distress', *BIS Quarterly Review, December* .

Habib, M. M. and Stracca, L. (2012), 'Getting beyond carry trade: What makes a safe haven currency?', *Journal of International Economics* **87**(1), 50–64.

Hakkio, C. S., Keeton, W. R. et al. (2009), 'Financial stress: What is it, how can it be measured, and why does it matter', *Economic Review* **94**(2), 5–50.

Hamilton, J. D. (1990), 'Analysis of time series subject to changes in regime', *Journal of econometrics* **45**(1-2), 39–70.

Hamilton, J. D. (1994), *Time series analysis*, Princeton university press.

Hansen, B. (2022), *Econometrics*, Princeton University Press.

Hoque, A. (2012), 'The effects of the european sovereign debt crisis on major currency markets', *International Research Journal of Finance and Economics* **101**, 75–80.

Hossfeld, O. and MacDonald, R. (2015), 'Carry funding and safe haven currencies: A threshold regression approach', *Journal of International Money and Finance* **59**, 185–202.

Joe, H. (1997), *Multivariate models and multivariate dependence concepts*, Chapman and Hall/CRC.

Joe, H. (2005), 'Asymptotic efficiency of the two-stage estimation method for copula-based models', *Journal of multivariate Analysis* **94**(2), 401–419.

Joe, H. (2014), *Dependence modeling with copulas*, CRC press.

Joe, H. and Xu, J. J. (1996), 'The estimation method of inference functions for margins for multivariate models', *Faculty Research and Publications* **NA**.

Kim, C.-J. (1994), 'Dynamic linear models with markov-switching', *Journal of Econometrics* **60**(1-2), 1–22.

Kim, C.-J., Nelson, C. R. et al. (1999), 'State-space models with regime switching: classical and gibbs-sampling approaches with applications', *MIT Press Books* **1**.

Kugler, P. and Weder, B. (2004), 'International portfolio holdings and swiss franc asset returns', *Swiss Journal of Economics and Statistics* **140**(III), 301–325.

Lee, K.-S. (2017), 'Safe-haven currency: An empirical identification', *Review of International Economics* **25**(4), 924–947.

Martin, I. (2017), 'What is the expected return on the market?', *The Quarterly Journal of Economics* **132**(1), 367–433.

Martin, V., Hurn, S. and Harris, D. (2013), *Econometric modelling with time series: specification, estimation and testing*, Cambridge University Press.

McCauley, R. N. and McGuire, P. (2009), 'Dollar appreciation in 2008: safe haven, carry trades, dollar shortage and overhedging', *BIS Quarterly Review* **December**, 86–92.

McNeil, A. J., Frey, R. and Embrechts, P. (2015), *Quantitative risk management: concepts, techniques and tools-revised edition*, Princeton university press.

Menkhoff, L., Sarno, L., Schmeling, M. and Schrimpf, A. (2012), 'Carry trades and global foreign exchange volatility', *The Journal of Finance* **67**(2), 681–718.

Nelsen, R. B. (2006), *An introduction to copulas*, Springer.

Nelson, D. B. (1991), 'Conditional heteroskedasticity in asset returns: A new approach', *Econometrica: Journal of the econometric society* pp. 347–370.

Pan, J. and Singleton, K. J. (2008), 'Default and recovery implicit in the term structure of sovereign cds spreads', *The Journal of Finance* **63**(5), 2345–2384.

Patton, A. J. (2009), Copula–based models for financial time series, *in* 'Handbook of financial time series', Springer, pp. 767–785.

Prasad, E. S. (2015), The dollar trap, *in* 'The Dollar Trap', Princeton University Press.

Ranaldo, A. and Söderlind, P. (2010), 'Safe haven currencies', *Review of finance* **14**(3), 385–407.

Reboredo, J. C. (2013), 'Is gold a safe haven or a hedge for the us dollar? implications for risk management', *Journal of Banking & Finance* **37**(8), 2665–2676.

Rey, H. (2015), Dilemma not trilemma: the global financial cycle and monetary policy independence, Technical report, National Bureau of Economic Research.

Rodriguez, J. C. (2007), 'Measuring financial contagion: A copula approach', *Journal of empirical finance* **14**(3), 401–423.

Schmidt, R. and Stadtmüller, U. (2006), 'Non-parametric estimation of tail dependence', *Scandinavian journal of statistics* **33**(2), 307–335.

Sklar, M. (1959), 'Fonctions de repartition an dimensions et leurs marges', *Publ. inst. statist. univ. Paris* **8**, 229–231.

Tachibana, M. (2018), 'Safe-haven and hedge currencies for the us, uk, and euro area stock markets: A copula-based approach', *Global Finance Journal* **35**, 82–96.

Wang, Y.-C., Wu, J.-L. and Lai, Y.-H. (2013), 'A revisit to the dependence structure between the stock and foreign exchange markets: A dependence-switching copula approach', *Journal of Banking & Finance* **37**(5), 1706–1719.

Wang, Y.-C., Wu, J.-L. and Lai, Y.-H. (2018), 'New evidence on asymmetric return–volume dependence and extreme movements', *Journal of Empirical Finance* **45**, 212–227.

Whaley, R. E. (2009), 'Understanding the vix', *The Journal of Portfolio Management* **35**(3), 98–105.

Wong, A. Y.-T. and Fong, T. P. W. (2018), 'Safehavenness of currencies', *The European Journal of Finance* **24**(4), 300–332.

Wooldridge, J. M. (2010), *Econometric analysis of cross section and panel data*, MIT press.

Yee, T. W. (2015), *Vector generalized linear and additive models: with an implementation in R*, Vol. 10, Springer.